

YEAR 2 MECHANICS • CHAPTER 1

Moments

Specification: Edexcel Paper 3 §9.1 — Moments • AQA section S1

KEY FACTS — THIS CHAPTER

- The **moment** of a force about a point is the force times the **perpendicular** distance from the point to the line of the force. Its unit is the newton metre (N m), and it turns either clockwise or anticlockwise.
- A force at an angle θ to a rod, applied a distance d along it, has moment $Fd \sin \theta$ about the other end.
- A **rigid body is in equilibrium** when the resultant force is zero **and** the resultant moment about every point is zero.
- Take moments about a point where an unknown force acts: that force has no moment there, so it drops out of the equation.
- A **uniform** rod's weight acts at its midpoint. A **non-uniform** rod's weight acts at its **centre of mass**, which you may need to find.
- A beam **on the point of tilting** about one support has **zero** reaction at the other.

Year 1 treated every body as a particle, a single point where all the forces act. A particle can speed up or slow down, but it can't turn. Real objects can: a seesaw rocks, a door swings, a shelf tips off its brackets. This chapter models them as **rigid bodies**, which keep their shape but can rotate, and asks what it takes to stop them turning.

1. The moment of a force

Push a door near the hinge and it barely moves. Push the same amount near the handle and it swings easily. The turning effect depends on the force **and** on how far from the turning point it acts.

THE MOMENT OF A FORCE

The moment of a force F about a point P is

$$\text{moment} = F \times d$$

where d is the **perpendicular** distance from P to the line of action of the force. It is measured in newton metres (N m) and has a sense: **clockwise** or **anticlockwise**.

To find the resultant moment of several forces, choose one sense as positive, then add the moments that turn that way and subtract the rest.

WORKED EXAMPLE 1.1 • THE RESULTANT MOMENT

Three forces act on a light horizontal rod: 6 N vertically down at a point 2 m to the left of P , 4 N vertically down 3 m to the right of P , and 5 N vertically up 1 m to the right of P . Find the resultant moment about P .

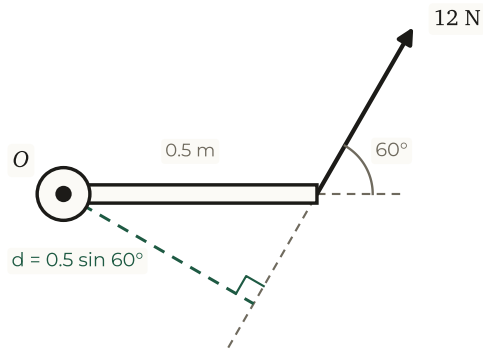
1. The 6 N force pulls down on the left, so it turns the rod anticlockwise. The 4 N force pulls down on the right: clockwise. The 5 N force pushes up on the right: anticlockwise.
2. Taking anticlockwise as positive:

$$\begin{aligned} 6 \times 2 - 4 \times 3 + 5 \times 1 \\ = 12 - 12 + 5 = 5 \end{aligned}$$

IN CONTEXT

The resultant moment about P is 5 N m anticlockwise.

When a force is not perpendicular to the rod, only its perpendicular component turns it. Equivalently, use the perpendicular distance to the line of the force. Both routes give the same product.



Only the perpendicular distance counts: $d = 0.5 \sin 60^\circ$.

WORKED EXAMPLE 1.2 • A FORCE AT AN ANGLE

A force of 12 N is applied to the end of a spanner of length 0.5 m, at 60° to the handle. Find its moment about the nut.

The perpendicular distance from the nut to the line of the force is $0.5 \sin 60^\circ$, so

$$\begin{aligned}\text{moment} &= 12 \times 0.5 \sin 60^\circ \\ &= 3\sqrt{3} = 5.20 \text{ N m (3 s.f.)}\end{aligned}$$

Resolving instead gives the same thing: the component of the force perpendicular to the handle is $12 \sin 60^\circ$, acting 0.5 m from the nut.

COMMON MISTAKE

Don't multiply by the distance **along** the rod when the force is at an angle to it. A force that points straight at the pivot has no turning effect at all, however far away it acts.

2. Rigid bodies in equilibrium

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium when

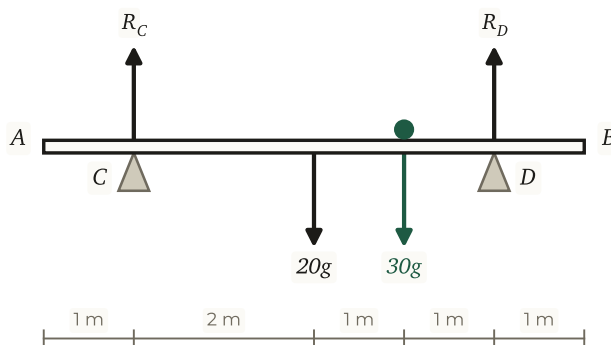
1. the **resultant force** on it is zero (it doesn't start moving), and
2. the **resultant moment** about any point is zero (it doesn't start turning).

WHY IT WORKS

The first condition is Newton's first law, as in Year 1. The second is new, and it doesn't matter which point you take moments about: if the forces balance, the resultant moment is the same about every point. So choose the point that makes the algebra easiest, usually one where an unknown force acts.

MODELLING ASSUMPTION

Modelling a plank as a **rigid rod** means it doesn't bend. Modelling it as **uniform** means its mass is spread evenly, so its weight acts at its midpoint. Modelling a person on it as a **particle** means their weight acts at a single point.



Example 1.3: every force on the plank, with the distances from A.

WORKED EXAMPLE 1.3 • A PLANK ON TWO SUPPORTS

A uniform plank AB of length 6 m and mass 20 kg rests horizontally on supports at C and D , where $AC = 1\text{ m}$ and $DB = 1\text{ m}$. A child of mass 30 kg stands on the plank 2 m from B . Find the reactions at the supports.

1. Take moments about C , so R_C drops out. The plank's weight acts 2 m from C , the child 3 m from C , and R_D acts 4 m from C :

$$4R_D = 20g \times 2 + 30g \times 3 = 130g$$

$$R_D = 32.5g = 318.5 \text{ N}$$

2. Resolve vertically: $R_C + R_D = 50g$, so $R_C = 17.5g = 171.5 \text{ N}$.

IN CONTEXT

The support at C pushes up with 171.5 N and the support at D with 318.5 N: the child stands closer to D , so D takes more of the load.

To check, take moments about D instead. Every other force acts to the left of D : R_C at 4 m, the plank's weight at 2 m and the child at 1 m. R_C turns the plank one way and both weights turn it the other, so $4R_C = 20g \times 2 + 30g \times 1 = 70g$ and $R_C = 17.5g$ again. Getting the side of each force right is most of the work.

3. Centres of mass

A **non-uniform** rod is heavier at one end, so its weight doesn't act at the midpoint. It acts at the rod's **centre of mass**, and the conditions for equilibrium can find where that is.

WORKED EXAMPLE 1.4 • FINDING THE CENTRE OF MASS

A non-uniform rod AB of length 3 m and mass 8 kg hangs horizontally from two vertical strings, one at A and one at B . The tension in the string at A is twice the tension in the string at B . Find the distance of the centre of mass from A .

1. Resolve vertically: $T_A + T_B = 8g$ and $T_A = 2T_B$, so $T_B = \frac{8}{3}g$.
2. Let the centre of mass be x m from A . Take moments about A :

$$8g \times x = T_B \times 3 = 8g$$

so $x = 1$.

IN CONTEXT

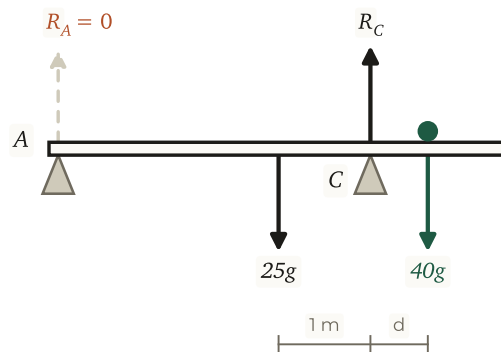
The centre of mass is 1 m from A : the rod is heavier towards A , which is why that string carries more of the weight.

4. On the point of tilting

Walk out along a plank that overhangs its support and, at some point, it tips up. Just before that happens, the plank is about to lift off the other support, so the reaction there has fallen to zero.

ON THE POINT OF TILTING

When a beam is **on the point of tilting** about one support, the reaction at the **other** support is **zero**. Take moments about the support it tilts about.



Example 1.5: on the point of tilting about C , the reaction at A is zero.

WORKED EXAMPLE 1.5 • HOW FAR CAN HE WALK?

A uniform plank of length 5 m and mass 25 kg rests horizontally on two supports, one at its end A and one at C , where $AC = 3.5$ m. A man of mass 40 kg walks along the plank from A towards the other end. How far beyond C can he go before the plank tilts?

1. On the point of tilting about C , the reaction at A is zero.
2. The plank's weight acts at its midpoint, 2.5 m from A , which is 1 m on the A side of C . Let the

man be d m beyond C . Taking moments about C :

$$25g \times 1 = 40g \times d$$

$$d = 0.625$$

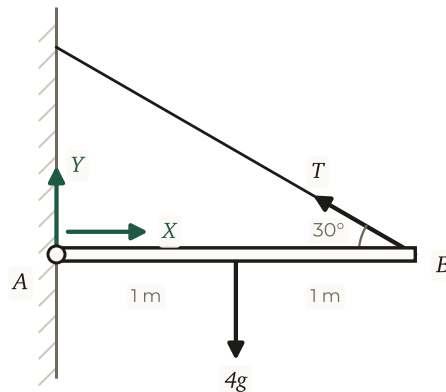
3. The plank overhangs C by 1.5 m, so he reaches this point before the end.

IN CONTEXT

He can walk 0.625 m beyond C . Any further and the plank tilts about C .

5. Forces at an angle

A shelf bracket, a drawbridge or a sign hanging from a pole: a rod hinged at one end and held up by a slanting string. The string pulls at an angle, so only its perpendicular component turns the rod, and the hinge supplies whatever force is needed to keep the rest in balance.



Example 1.6: the string, the weight and the two components of the hinge force.

WORKED EXAMPLE 1.6 • A ROD HELD BY A STRING

A uniform rod AB of length 2 m and mass 4 kg is hinged to a vertical wall at A . It is held horizontal by a light string attached to B and to a point on the wall above A , and the string makes an angle of 30° with the rod. Find the tension in the string and the force exerted by the hinge.

1. Take moments about A , so the unknown hinge force drops out. The component of the tension

perpendicular to the rod is $T \sin 30^\circ$, acting 2 m from A :

$$T \sin 30^\circ \times 2 = 4g \times 1$$

$$T = 4g = 39.2 \text{ N}$$

2. Resolve for the hinge force, with components X horizontally and Y vertically:

$$X = T \cos 30^\circ = 2\sqrt{3}g$$

$$= 33.9 \text{ N}$$

$$Y = 4g - T \sin 30^\circ$$

$$= 2g = 19.6 \text{ N}$$

3. Its size is $\sqrt{X^2 + Y^2} = \sqrt{12g^2 + 4g^2} = 4g = 39.2 \text{ N}$, at an angle $\tan^{-1} \frac{Y}{X} = 30^\circ$ above the horizontal.

IN CONTEXT

The string pulls with 39.2 N. The hinge pushes on the rod with 39.2 N at 30° above the horizontal, out from the wall.

COMMON MISTAKE

A hinge can push or pull in any direction, so give its force two unknown components. Don't assume it acts along the rod.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution. The solutions show the full method, not just answers. Take $g = 9.8 \text{ m s}^{-2}$ throughout. Rods and planks are rigid, and strings are light, unless a question says otherwise.

Warm-up

Question 1

[2 marks]

Find the moment about P of

- (a) a force of 15 N whose line of action is a perpendicular distance of 0.4 m from P ,
- (b) a force of 20 N acting at the end of a rod 3 m long, pivoted at P , at 30° to the rod.

SOLUTION

(a) $15 \times 0.4 = 6 \text{ N m}$.

(b) The perpendicular distance is $3 \sin 30^\circ = 1.5 \text{ m}$, so the moment is $20 \times 1.5 = 30 \text{ N m}$.

Question 2

[3 marks]

A light rod AB of length 3 m rests on a pivot at P , where $AP = 1 \text{ m}$. A weight of 30 N hangs from A . Find the vertical force that must act at B to keep the rod horizontal, and the force on the pivot.

SOLUTION

Take moments about P : $30 \times 1 = F \times 2$, so $F = 15 \text{ N}$, acting vertically **downwards** at B .

Resolve vertically: the pivot pushes up with $30 + 15 = 45 \text{ N}$.

Question 3

[3 marks]

Three vertical forces act on a light horizontal rod AC of length 5 m: 8 N down at A , 5 N up at B , where $AB = 2 \text{ m}$, and 10 N down at C . Find the resultant moment about B .

SOLUTION

The 5 N force acts at B , so it has no moment about B . The 8 N force acts 2 m to the left of B (anticlockwise, 16 N m); the 10 N force acts 3 m to the right of B (clockwise, 30 N m).

The resultant moment is $30 - 16 = 14 \text{ N m}$ clockwise.

Question 4

[4 marks]

A uniform rod AB of length 4 m and mass 6 kg rests horizontally on supports at A and B . A particle of mass 2 kg is placed on the rod 1 m from A . Find the reactions at the supports.

SOLUTION

Take moments about A :

$$4R_B = 6g \times 2 + 2g \times 1 = 14g$$

$$R_B = 3.5g = 34.3 \text{ N}$$

Resolve vertically: $R_A = 8g - 3.5g = 4.5g = 44.1 \text{ N}$.

Standard**Question 5**

[5 marks]

A uniform plank AB of length 8 m and mass 30 kg rests horizontally on supports at C and D , where $AC = 1$ m and $DB = 2$ m. A child of mass 40 kg stands on the plank at E , and the reactions at the two supports are equal. Find the distance AE .

SOLUTION

The total weight is $70g$, so each reaction is $35g$. $CD = 5$ m, the plank's weight acts 3 m from C , and the child is $AE - 1$ from C . Taking moments about C :

$$\begin{aligned} 35g \times 5 &= 30g \times 3 + 40g(AE - 1) \\ 85 &= 40(AE - 1) \end{aligned}$$

so $AE - 1 = 2.125$ and $AE = 3.125$ m.

Question 6

[4 marks]

A non-uniform rod AB of length 4 m and mass 12 kg rests horizontally on supports at A and B . The reaction at A is $7g$ N. Find the distance of the centre of mass of the rod from A .

SOLUTION

Resolve vertically: $R_B = 12g - 7g = 5g$.

Take moments about A , with the centre of mass x m from A : $12g \times x = 5g \times 4$, so $x = \frac{5}{3} = 1.67$ m (3 s.f.).

Question 7

[5 marks]

A uniform plank AB of length 6 m and mass 20 kg rests horizontally on two supports, one at A and one at C , where $AC = 4$ m.

- (a) Find the greatest mass that can be placed at B without the plank tilting. [3]
- (b) A particle of mass 6 kg is placed at B . Find the reactions at A and C . [2]

SOLUTION

(a) On the point of tilting about C , the reaction at A is zero. The plank's weight acts 1 m on the A side of C , and B is 2 m on the other side:

$$20g \times 1 = Mg \times 2 \quad \Rightarrow \quad M = 10$$

The greatest mass is 10 kg.

(b) Take moments about A : $4R_C = 20g \times 3 + 6g \times 6 = 96g$, so $R_C = 24g = 235.2$ N. Then $R_A = 26g - 24g = 2g = 19.6$ N.

Question 8

[5 marks]

A uniform rod AB of length 2 m and mass 5 kg is hinged to a vertical wall at A . It is held horizontal by a light string attached to B and to a point on the wall above A , making an angle of 40° with the rod.

(a) Find the tension in the string. [3]

(b) Show that the force exerted by the hinge has the same size as the tension. [2]

SOLUTION

(a) Take moments about A :

$$\begin{aligned} T \sin 40^\circ \times 2 &= 5g \times 1 \\ T &= \frac{5g}{2 \sin 40^\circ} \\ &= 38.1 \text{ N (3 s.f.)} \end{aligned}$$

(b) From (a), $T \sin 40^\circ = 2.5g$. The hinge's components are $X = T \cos 40^\circ$ horizontally and $Y = 5g - T \sin 40^\circ = 2.5g = T \sin 40^\circ$ vertically, so

$$\begin{aligned} \sqrt{X^2 + Y^2} \\ &= T \sqrt{\cos^2 40^\circ + \sin^2 40^\circ} = T \end{aligned}$$

Question 9

[4 marks]

A uniform rod AB of length 1 m and mass 2 kg has a particle of mass 3 kg fixed at A and a particle of mass 5 kg fixed at B . Find the distance from A of the point where the rod balances on a single support.

SOLUTION

Let the support be x m from A . The rod's weight acts $0.5 - x$ from it and the 5 kg particle $1 - x$ from it, both on the B side:

$$\begin{aligned} 3g \times x &= 2g(0.5 - x) + 5g(1 - x) \\ 10x &= 6 \end{aligned}$$

so the rod balances 0.6 m from A .

Exam style

Question 10

[8 marks]

A uniform beam AB of length 5 m and mass 40 kg rests horizontally on supports at C and D , where $AC = 0.5$ m and $DB = 1$ m. A man of mass 80 kg stands on the beam.

- (a) The reactions at C and D are equal. Find the man's distance from A . [4]
- (b) The man walks towards B . How far beyond D can he go before the beam tilts? [3]
- (c) State one modelling assumption you have made about the man. [1]

SOLUTION

(a) Each reaction is $\frac{120g}{2} = 60g$. Let the man be x m from A . $CD = 3.5$ m and the beam's weight acts 2 m from C . Taking moments about C :

$$\begin{aligned} 60g \times 3.5 &= 40g \times 2 + 80g(x - 0.5) \\ 130 &= 80(x - 0.5) \end{aligned}$$

so $x = 0.5 + 1.625 = 2.125$ m.

(b) On the point of tilting about D , the reaction at C is zero. The beam's weight acts 1.5 m on the A side of D . With the man d m beyond D : $40g \times 1.5 = 80g \times d$, so $d = 0.75$ m. That is less than $DB = 1$ m, so he reaches it: he can go 0.75 m beyond D .

(c) The man is modelled as a **particle**, so his weight acts at a single point.

Question 11

[8 marks]

A non-uniform rod AB of length 6 m and mass 10 kg hangs horizontally from two vertical strings attached at C and D , where $AC = 1$ m and $DB = 1$ m. A particle of mass 5 kg is attached to the rod at A , and the tensions in the two strings are then equal.

- (a) Find the distance of the centre of mass of the rod from A . [5]
- (b) The particle is removed. Find the new tensions in the strings. [3]

SOLUTION

(a) Each tension is $\frac{15g}{2} = 7.5g$. Let the centre of mass be x m from A . The particle at A is 1 m on one side of C ; D and the centre of mass are on the other. Taking moments about C :

$$\begin{aligned} 7.5g \times 4 + 5g \times 1 &= 10g(x - 1) \\ 35 &= 10(x - 1) \end{aligned}$$

so $x = 4.5$ m.

(b) Take moments about C : $T_D \times 4 = 10g \times 3.5$, so $T_D = 8.75g = 85.75$ N. Then $T_C = 10g - 8.75g = 1.25g = 12.25$ N.

Question 12

[8 marks]

A uniform rod AB of length 3 m and mass 6 kg is hinged to a vertical wall at A , and a particle of mass 4 kg is attached at B . The rod is held horizontal by a light string attached at C , where $AC = 2$ m, and to a point on the wall above A . The string makes an angle of 30° with the rod.

- (a) Find the tension in the string. [3]
- (b) Find the horizontal and vertical components of the force exerted by the hinge, and describe its direction. [5]

SOLUTION

(a) Take moments about A :

$$\begin{aligned} T \sin 30^\circ \times 2 &= 6g \times 1.5 + 4g \times 3 \\ &= 21g \end{aligned}$$

so $T = 21g = 205.8$ N, or 206 N (3 s.f.).

(b) Horizontally the hinge balances the pull of the string towards the wall: $X = T \cos 30^\circ = 178.2 \dots$, so 178 N (3 s.f.) out from the wall.

Vertically, taking Y upwards: $Y + T \sin 30^\circ = 10g$, so $Y = 10g - 10.5g = -0.5g = -4.9$ N.

The vertical component is negative: the hinge pulls the rod **down** with 4.9 N. The string holds up slightly more than the total weight, because it is attached nearer the wall than the load at B .