

YEAR 2 MECHANICS · CHAPTER 2

Forces and Friction

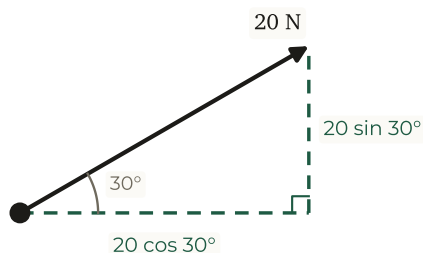
Specification: Edexcel Paper 3 §8.2, 8.4–8.6 — Forces and Newton's laws · AQA section R2, R4–R6

KEY FACTS — THIS CHAPTER

- A force F at an angle θ to a direction has component $F \cos \theta$ **along** that direction and $F \sin \theta$ **perpendicular** to it.
- To find a resultant, add the components in two perpendicular directions: magnitude $\sqrt{X^2 + Y^2}$, direction $\tan^{-1} \frac{Y}{X}$ from the X direction.
- A particle is in **equilibrium** when the forces on it balance in **two** perpendicular directions.
- **Friction** opposes motion, or the tendency to move. $F \leq \mu R$, where μ is the **coefficient of friction** and R the normal reaction. When the body moves or is **on the point of moving**, $F = \mu R$ (limiting friction).
- On a slope at angle α , the weight mg splits into $mg \sin \alpha$ **down** the slope and $mg \cos \alpha$ **into** it.
- A particle rests on a rough slope, with no other forces along it, exactly when $\tan \alpha \leq \mu$.

Year 1 kept every force along one of two perpendicular lines: horizontal and vertical, or along a string. Real forces pull at angles, slopes tilt the weight out of line, and surfaces grip. This chapter adds the tools for all three: resolving forces, the friction model $F \leq \mu R$, and particles on inclined planes.

1. Resolving forces



A force and its two components: the adjacent side gets the cosine.

RESOLVING A FORCE

A force F making an angle θ with a direction has components

$F \cos \theta$ along that direction

$F \sin \theta$ perpendicular to it

Together the two components have exactly the same effect as F .

WORKED EXAMPLE 2.1 · COMPONENTS OF A FORCE

A force of 20 N acts at 30° above the horizontal. Find its horizontal and vertical components.

$$\text{horizontal: } 20 \cos 30^\circ = 17.3 \text{ N (3 s.f.)}$$

$$\text{vertical: } 20 \sin 30^\circ = 10 \text{ N}$$

To combine several forces, resolve each one in the same two directions, add, and then put the two totals back together with Pythagoras.

WORKED EXAMPLE 2.2 · THE RESULTANT OF THREE FORCES

Three forces act on a particle: 10 N due east, 6 N due north, and 8 N in the direction 45° west of north. Find the magnitude and direction of the resultant.

1. Resolve east and north:

$$\text{east: } 10 - 8 \sin 45^\circ = 4.343 \dots$$

$$\text{north: } 6 + 8 \cos 45^\circ = 11.656 \dots$$

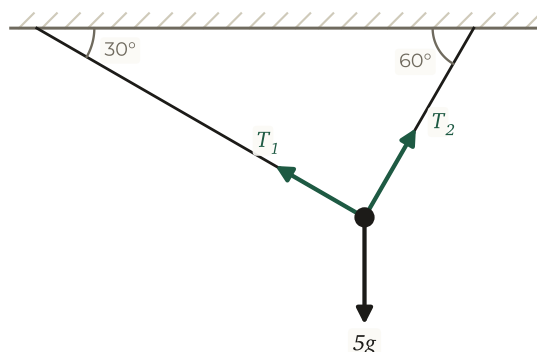
2. The magnitude is $\sqrt{4.343 \dots^2 + 11.656 \dots^2} = 12.4 \text{ N (3 s.f.)}$, and the angle east of north is $\tan^{-1} \frac{4.343 \dots}{11.656 \dots} = 20.4^\circ$.

IN CONTEXT

The resultant is 12.4 N on a bearing of 020° (to the nearest degree).

2. Particles in equilibrium

A particle is in equilibrium when the resultant force is zero. That is two equations: the forces balance in each of two perpendicular directions. Choose the directions that make the equations simplest, usually horizontal and vertical, or along and perpendicular to a slope.



Example 2.3: two tensions and the weight, in equilibrium.

WORKED EXAMPLE 2.3 • HANGING FROM TWO STRINGS

A particle of mass 5 kg hangs in equilibrium from two light strings. One makes an angle of 30° with the horizontal and has tension T_1 ; the other makes 60° with the horizontal and has tension T_2 . Find both tensions.

1. Resolve horizontally and vertically:

$$T_1 \cos 30^\circ = T_2 \cos 60^\circ$$

$$5g = T_1 \sin 30^\circ + T_2 \sin 60^\circ$$

2. The first gives $T_2 = \sqrt{3}T_1$. Substituting, $\frac{1}{2}T_1 + \frac{3}{2}T_1 = 5g$, so $T_1 = 2.5g = 24.5 \text{ N}$ and $T_2 = 2.5\sqrt{3}g = 42.4 \text{ N}$ (3 s.f.).

IN CONTEXT

The steeper string does more of the holding up: 42.4 N against 24.5 N.

3. Friction

Push a heavy box gently and nothing happens: friction pushes back just as hard. Push harder and friction grows to match, but only up to a limit. Beyond it, the box slides.

THE FRICTION MODEL

Friction acts along the surface, opposing motion or the tendency to move, and

$$F \leq \mu R$$

where R is the normal reaction and μ is the **coefficient of friction**, a number that depends on the two surfaces. A **smooth** surface has $\mu = 0$.

When the body is **moving**, or **on the point of moving**, friction is limiting: $F = \mu R$.

WORKED EXAMPLE 2.4 · WILL IT MOVE?

A box of mass 10 kg rests on a rough horizontal floor, and the coefficient of friction is 0.3. A horizontal force P acts on it. Describe what happens when (a) $P = 20$ N and (b) $P = 40$ N.

$R = 10g = 98$ N, so the greatest friction available is $0.3 \times 98 = 29.4$ N.

(a) $20 < 29.4$, so friction can match the push: the box stays at rest, with friction 20 N.

(b) $40 > 29.4$, so the box slides, with friction 29.4 N:

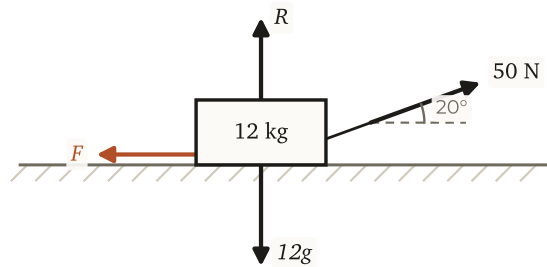
$$40 - 29.4 = 10a$$

$$a = 1.06 \text{ m s}^{-2}$$

COMMON MISTAKE

Friction is not always μR . It is only as large as it needs to be, up to μR . In Example 2.4(a) it is 20 N, not 29.4 N: if it were 29.4 N, the box would accelerate backwards.

When a force pulls at an angle, it also lifts: the normal reaction, and with it the friction, both fall.



Example 2.5: the rope lifts as well as pulls, so R is less than $12g$.

WORKED EXAMPLE 2.5 · PULLING AT AN ANGLE

A sledge of mass 12 kg is pulled across rough horizontal ground by a rope with tension 50 N at 20° above the horizontal. The coefficient of friction is 0.25 . Find the acceleration of the sledge.

1. Resolve vertically: $R + 50 \sin 20^\circ = 12g$, so $R = 117.6 - 17.10 \dots = 100.49 \dots\text{ N}$.
2. The sledge moves, so $F = 0.25R = 25.12 \dots\text{ N}$.
3. Resolve horizontally:

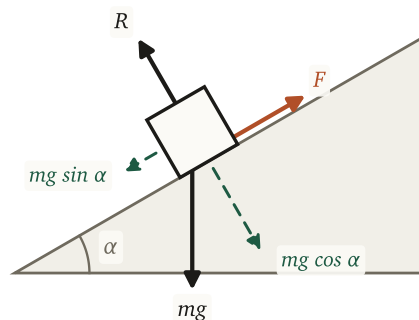
$$50 \cos 20^\circ - F = 12a$$

$$a = 1.82\text{ m s}^{-2}$$

to 3 significant figures.

4. Inclined planes

On a slope, resolve **along** and **perpendicular to** the slope. The normal reaction and friction then each lie along one direction, and only the weight needs splitting.



The weight splits into $mg \sin \alpha$ down the slope and $mg \cos \alpha$ into it.

A PARTICLE ON A SLOPE

On a plane inclined at α to the horizontal, the weight mg has components

$mg \sin \alpha$ down the slope

$mg \cos \alpha$ into the slope

With no other forces perpendicular to the slope, $R = mg \cos \alpha$.

WHY IT WORKS

The angle between the weight and the perpendicular to the slope is α too: tilt the slope by α and its perpendicular tilts by the same angle away from the vertical. So the component into the slope is the adjacent side, $mg \cos \alpha$, and the component down it is $mg \sin \alpha$.

WORKED EXAMPLE 2.6 · A SMOOTH SLOPE

A particle is released from rest on a smooth plane inclined at 30° to the horizontal. Find its acceleration and its speed after 2 s.

Resolving down the slope, $mg \sin 30^\circ = ma$, so $a = g \sin 30^\circ = 4.9 \text{ m s}^{-2}$. The mass cancels. After 2 s, $v = 0 + 4.9 \times 2 = 9.8 \text{ m s}^{-1}$.

The classic slope in exam questions has $\tan \alpha = \frac{3}{4}$, so $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$ exactly: the 3, 4, 5 right-angled triangle.

WORKED EXAMPLE 2.7 · DOES IT SLIDE?

A particle of mass 2 kg is placed on a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction is 0.5. Show that the particle slides, and find its acceleration.

1. Perpendicular to the slope: $R = 2g \cos \alpha = 1.6g = 15.68$ N, so the greatest friction is $0.5 \times 1.6g = 0.8g = 7.84$ N.
2. Down the slope the weight pulls with $2g \sin \alpha = 1.2g = 11.76$ N, more than friction can hold, so the particle slides.
3. Down the slope:

$$1.2g - 0.8g = 2a$$

$$a = 0.2g = 1.96 \text{ m s}^{-2}$$

Friction always opposes the way the body would move, so its direction depends on the other forces. Push up a slope and friction acts down it; hold a particle back from sliding and friction acts up it.

WORKED EXAMPLE 2.8 · FRICTION CHANGES DIRECTION

The particle of Example 2.7 is held on the slope by a force P acting up the slope, parallel to it. Find P when the particle is on the point of moving (a) up the slope and (b) down the slope.

- (a) About to move up: friction acts **down** the slope, so $P = 1.2g + 0.8g = 2g = 19.6$ N.
- (b) About to slide down: friction acts **up** the slope, so $P = 1.2g - 0.8g = 0.4g = 3.92$ N.

IN CONTEXT

Any force between 3.92 N and 19.6 N keeps the particle at rest. Below that it slides down; above it, it moves up.

COMMON MISTAKE

On a slope, R is **not** mg . It is $mg \cos \alpha$, and less again if another force pulls away from the slope. Every friction calculation starts from R , so find it first.

MODELLING ASSUMPTION

The block is modelled as a **particle**, so it can't topple, and μ is taken as **constant**, the same at rest and when sliding. In reality friction just before sliding is usually a little larger than once the body is moving.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution. The solutions show the full method, not just answers. Take $g = 9.8 \text{ m s}^{-2}$ throughout, and give answers to 3 significant figures where they aren't exact.

Warm-up**Question 1**

[2 marks]

A force of 15 N acts at 40° above the horizontal. Find its horizontal and vertical components.

SOLUTION

Horizontal: $15 \cos 40^\circ = 11.5 \text{ N}$. Vertical: $15 \sin 40^\circ = 9.64 \text{ N}$.

Question 2

[3 marks]

Forces of 7 N due east and 24 N due north act on a particle. Find the magnitude of the resultant and the angle it makes with east.

SOLUTION

The magnitude is $\sqrt{7^2 + 24^2} = 25 \text{ N}$, at $\tan^{-1} \frac{24}{7} = 73.7^\circ$ north of east.

Question 3

[3 marks]

A box of mass 4 kg rests on rough horizontal ground, where the coefficient of friction is 0.2. A horizontal force of 5 N acts on it. Does the box move? Find the friction force.

SOLUTION

$R = 4g = 39.2 \text{ N}$, so friction can be at most $0.2 \times 39.2 = 7.84 \text{ N}$. That is more than 5 N, so the box stays at rest, and friction is 5 N: just enough to balance the push.

Question 4

[2 marks]

A particle slides down a smooth plane inclined at 25° to the horizontal. Find its acceleration.

SOLUTION

$$a = g \sin 25^\circ = 4.14 \text{ m s}^{-2}.$$

Standard**Question 5**

[5 marks]

A particle is in equilibrium under three forces: 8 N due east, P N due north, and Q N in the direction 30° south of west. Find P and Q .

SOLUTION

Resolve east: $8 = Q \cos 30^\circ$, so $Q = \frac{8}{\cos 30^\circ} = 9.24 \text{ N}$.

Resolve north: $P = Q \sin 30^\circ = 4.62 \text{ N}$.

Question 6

[5 marks]

A particle of mass 3 kg hangs in equilibrium from two light strings, which make angles of 40° and 50° with the horizontal. Find the tension in each string.

SOLUTION

Let the tensions be T_1 (at 40°) and T_2 (at 50°). Resolving:

$$T_1 \cos 40^\circ = T_2 \cos 50^\circ$$

$$T_1 \sin 40^\circ + T_2 \sin 50^\circ = 3g$$

The first gives $T_2 = T_1 \frac{\cos 40^\circ}{\cos 50^\circ}$. Substituting and solving, $T_1 = 18.9 \text{ N}$ and $T_2 = 22.5 \text{ N}$.

Question 7

[5 marks]

A sledge of mass 20 kg is pulled across rough horizontal snow at constant speed by a rope with tension 80 N at 30° above the horizontal. Find the coefficient of friction.

SOLUTION

Constant speed, so the forces balance.

Vertically: $R = 20g - 80 \sin 30^\circ = 156 \text{ N}$.

Horizontally: $F = 80 \cos 30^\circ = 69.28 \dots \text{ N}$.

It is moving, so $F = \mu R$: $\mu = \frac{69.28 \dots}{156} = 0.444$.

Question 8

[4 marks]

A block of mass 5 kg rests on a rough plane inclined at 20° to the horizontal. Find the friction force, and the least possible value of the coefficient of friction.

SOLUTION

Along the slope, friction balances the component of the weight: $F = 5g \sin 20^\circ = 16.8 \text{ N}$.

It stays at rest, so $F \leq \mu R$ with $R = 5g \cos 20^\circ$. That needs $\mu \geq \tan 20^\circ = 0.364$.

Question 9

[5 marks]

A particle of mass 4 kg is released from rest on a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction is 0.25. Find its acceleration and how far it slides in the first 2 s.

SOLUTION

$R = 4g \times \frac{4}{5}$, so the friction is $0.25 \times \frac{16}{5}g = 0.8g$. Down the slope:

$$4g \times \frac{3}{5} - 0.8g = 4a$$

$$a = 0.4g = 3.92 \text{ m s}^{-2}$$

In 2 s it slides $s = \frac{1}{2} \times 3.92 \times 2^2 = 7.84 \text{ m}$.

Exam style

Question 10

[8 marks]

A particle of mass 2 kg rests on a rough plane inclined at 30° to the horizontal. The coefficient of friction is 0.4. A force $P \text{ N}$ acts on the particle up the slope, parallel to it.

- Find the value of P when the particle is on the point of moving up the slope. [3]
- Find the least value of P that stops it sliding down. [2]
- Given that $P = 25$, find the acceleration of the particle. [3]

SOLUTION

$R = 2g \cos 30^\circ$, so limiting friction is $0.4 \times 2g \cos 30^\circ = 6.789 \dots$ N. The weight's component down the slope is $2g \sin 30^\circ = 9.8$ N.

(a) Friction acts down the slope: $P = 9.8 + 6.789 \dots = 16.6$ N.

(b) Friction acts up the slope: $P = 9.8 - 6.789 \dots = 3.01$ N.

(c) $25 > 16.6$, so the particle moves up the slope with limiting friction acting down it:

$$25 - 9.8 - 6.789 \dots = 2a$$

$$a = 4.21 \text{ m s}^{-2}$$

Question 11

[7 marks]

A box of mass 8 kg is pushed across a rough horizontal floor by a force of 60 N directed at 25° **below** the horizontal. The coefficient of friction is 0.35.

(a) Find the acceleration of the box. [5]

(b) Explain why pushing downwards like this makes the friction larger. [2]

SOLUTION

(a) Vertically the push adds to the weight: $R = 8g + 60 \sin 25^\circ = 103.75 \dots$ N, so $F = 0.35R = 36.31 \dots$ N. Horizontally:

$$60 \cos 25^\circ - 36.31 \dots = 8a$$

$$a = 2.26 \text{ m s}^{-2}$$

(b) Part of the push presses the box into the floor, which increases the normal reaction, and the limiting friction μR grows with it.

Question 12

[9 marks]

A particle is projected with speed 6 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction is 0.5.

(a) Find how far up the plane the particle travels before coming to rest. [5]

(b) Show that it then slides back down, and find its speed when it returns to its starting point. [4]

SOLUTION

Per kilogram: the weight's component along the slope is $\frac{3}{5}g$ and the limiting friction is $0.5 \times \frac{4}{5}g = 0.4g$.

(a) Moving up, both act down the slope, so the deceleration is $0.6g + 0.4g = g$. From $v^2 = u^2 + 2as$: $0 = 36 - 2 \times 9.8 \times s$, so $s = 1.84$ m.

(b) At rest, friction could hold at most $0.4g$ per kilogram, less than the $0.6g$ pulling it down, so it slides back ($\tan \alpha = 0.75 > \mu$). Now friction acts up the slope and $a = 0.6g - 0.4g = 0.2g = 1.96 \text{ m s}^{-2}$.

$$v^2 = 2 \times 1.96 \times 1.836 \dots = 7.2$$

$$v = 2.68 \text{ m s}^{-1}$$