

YEAR 1 PURE · CHAPTER 3

Using Graphs

Specification: Edexcel Pure 2 — Algebra and functions · AQA section B

KEY FACTS — THIS CHAPTER

- $y = x^n$: **even** n gives a U-shaped curve symmetric about the y -axis; **odd** n runs from bottom-left to top-right through the origin.
- A factorised cubic **crosses** the x -axis at each single root and **touches** it at a repeated root.
- $y = \frac{k}{x}$ has two branches with the **axes as asymptotes** — lines the curve approaches but never reaches.
- Curves meet where their equations are **equal**: solve simultaneously, and the number of solutions is the number of intersection points.
- Transformations: $f(x) + a$ shifts **up** a ; $f(x + a)$ shifts **left** a ; $af(x)$ stretches vertically $\times a$; $f(ax)$ stretches horizontally $\times \frac{1}{a}$.
- Inside the bracket acts on x and does the **opposite** of what it looks like; outside acts on y and does exactly what it looks like.

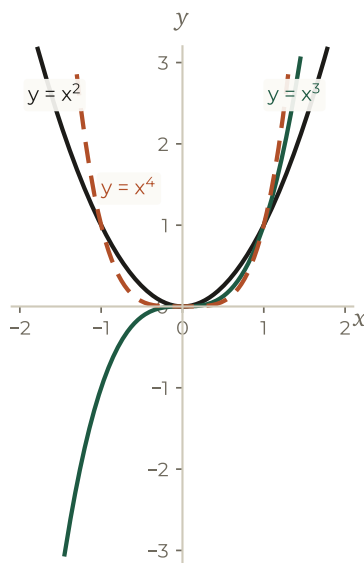
A sketch is the fastest piece of mathematics you can do: five seconds of curve drawing often replaces a page of algebra, and examiners ask for it constantly. This chapter builds your mental library of shapes — and then shows you how to slide, stretch and flip them.

1. Cubics, quartics and the shape of x^n

You already know the parabola. The rest of the x^n family follows a simple pattern, split by whether n is even or odd.

KEY RESULT

- **Even powers** ($x^2, x^4, \dots$): U-shaped, symmetric about the y -axis, never negative. Higher powers are flatter near the origin and steeper outside it.
- **Odd powers** ($x^3, x^5, \dots$): run from bottom-left to top-right through the origin, with rotational symmetry — negative inputs give negative outputs.

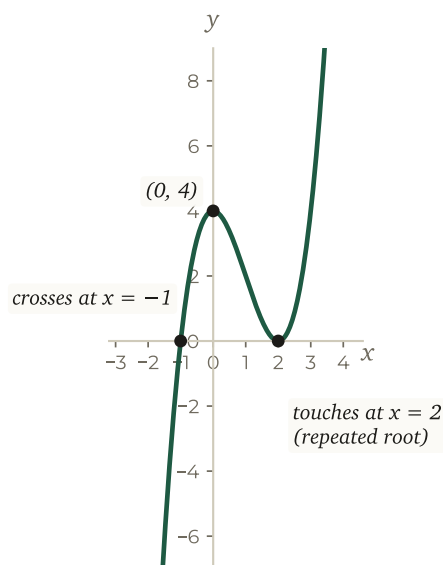


The x^n family near the origin — even powers make a U, odd powers climb through it.

For a **factorised cubic** like $y = (x + 1)(x - 2)^2$, the roots tell you almost everything:

SKETCHING A FACTORISED CUBIC

1. Each **single root** is a clean crossing of the x -axis.
2. Each **repeated root** (a squared bracket) is a point where the curve **touches** the axis and turns back — just like a parabola's vertex sitting on it.
3. The y -intercept comes from substituting $x = 0$.
4. For large x the highest power wins: a positive- x^3 cubic ends up in the top right, a negative one in the bottom right.



$y = (x + 1)(x - 2)^2$ — a crossing at the single root, a touch at the repeated one.

WORKED EXAMPLE 3.1 • SKETCHING A CUBIC FROM FACTORISED FORM

Sketch the curve $y = (x + 1)(x - 2)^2$, marking the points where it meets the axes.

1. Roots from the factors: $x = -1$ (single — the curve crosses) and $x = 2$ (repeated — the curve touches and turns back).
2. The y -intercept, from $x = 0$:

$$y = (1)(-2)^2 = 4$$

3. The bracket expansion would start $x^3 + \dots$, so the curve falls to the bottom-left and climbs to the top-right.
4. Join the features: up through $(-1, 0)$, over the hump past $(0, 4)$, down to touch the axis at $(2, 0)$, then up and away.

COMMON MISTAKE

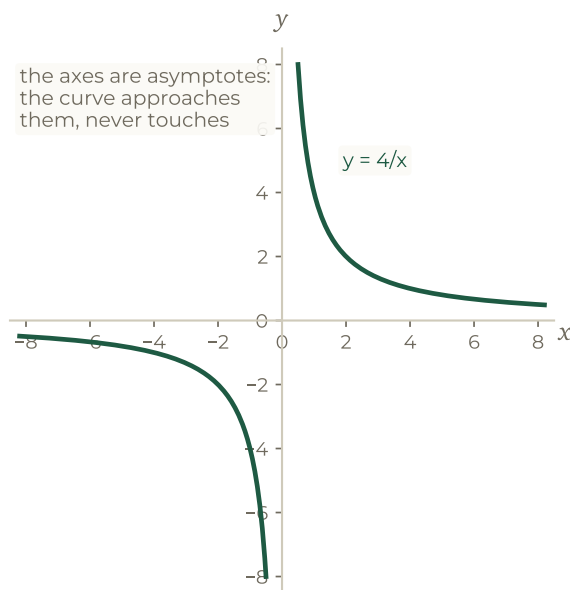
A repeated root is not a crossing. $(x - 2)^2$ keeps the same sign on both sides of $x = 2$, so the curve bounces off the axis there. Cross at single roots, **touch** at squared ones.

2. Reciprocal graphs

The curve $y = \frac{k}{x}$ behaves unlike any polynomial: it isn't defined at $x = 0$, and it comes in two separate branches.

KEY RESULT

$y = \frac{k}{x}$ (for $k > 0$) lives in the top-right and bottom-left quadrants; for $k < 0$, the other two. Both axes are **asymptotes**: as x grows huge, y shrinks towards zero; as x creeps towards zero, y blows up. The curve approaches the lines but never touches them. ($y = \frac{k}{x^2}$ is similar but, with its even power, keeps both branches on the same side of the x -axis.)



$y = 4/x$ — two branches, with both axes as asymptotes.

WORKED EXAMPLE 3.2 · PINNING DOWN A RECIPROCAL CURVE

The curve $y = \frac{k}{x}$ passes through the point $(2, 5)$. Find k , and the value of y when $x = -4$.

1. Substitute the known point:

$$5 = \frac{k}{2} \implies k = 10$$

2. Then at $x = -4$:

$$y = \frac{10}{-4} = -\frac{5}{2}$$

3. Where graphs meet

Two curves intersect where their y -values agree — so set the equations equal and solve. Every solution is an intersection point, and counting solutions counts crossings (which is exactly the discriminant trick from Chapter 2).

WORKED EXAMPLE 3.3 · INTERSECTION OF A LINE AND A CURVE

Find the points where the line $y = 2x - 1$ meets the curve $y = x^2 - 4x + 7$.

1. Set the right-hand sides equal:

$$x^2 - 4x + 7 = 2x - 1$$

2. Rearrange to zero and solve:

$$x^2 - 6x + 8 = (x - 2)(x - 4) = 0 \implies x = 2 \text{ or } x = 4$$

3. Substitute into the **line** for the y -values: $(2, 3)$ and $(4, 7)$.
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WORKED EXAMPLE 3.4 · SHOWING TWO GRAPHS NEVER MEET

Show that the line $y = 3x - 11$ does not meet the curve $y = x^2 - x - 6$.

1. If they met, the equations would agree:

$$x^2 - x - 6 = 3x - 11 \implies x^2 - 4x + 5 = 0$$

2. Check the discriminant:

$$\Delta = (-4)^2 - 4(1)(5) = 16 - 20 = -4 < 0$$

No real solutions, so there are no intersection points — the line passes clean underneath the parabola.



4. Transformations of graphs

One curve, four dials: you can slide a graph in either direction, stretch it, or flip it — and each move is a small edit to the equation.

THE FOUR TRANSFORMATIONS

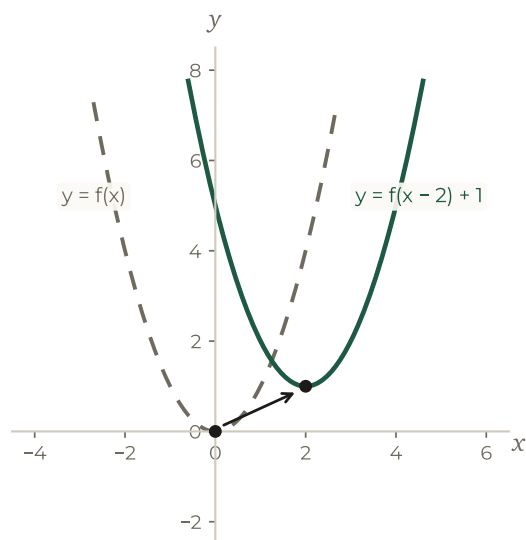
Starting from $y = f(x)$:

- $y = f(x) + a$ — translate **up** by a
- $y = f(x + a)$ — translate **left** by a
- $y = af(x)$ — stretch **vertically**, scale factor a
- $y = f(ax)$ — stretch **horizontally**, scale factor $\frac{1}{a}$

Negative values reflect: $y = -f(x)$ flips in the x -axis, $y = f(-x)$ flips in the y -axis.

WHY IT WORKS

Changes **outside** f happen after the function — they act on y , so they do exactly what they say. Changes **inside** the bracket happen to x *before* f sees it, so they work in reverse: $f(x + 3)$ asks “what input is 3 ahead of me?”, which drags the whole picture **left**. Inside the bracket, expect the opposite of what it looks like.



$y = f(x - 2) + 1$: the whole curve — vertex included — slides 2 right and 1 up.

WORKED EXAMPLE 3.5 · TRACKING A POINT THROUGH TRANSFORMATIONS

The graph of $y = f(x)$ has a maximum point at $(3, 5)$. Write down the coordinates of the maximum point of (a) $y = f(x + 2)$, (b) $y = 2f(x)$, (c) $y = f(2x)$.

1. **(a)** Inside the bracket, so it acts on x , in reverse: translate 2 **left**. Maximum at $(1, 5)$.
2. **(b)** Outside, acting on y : stretch vertically $\times 2$. Maximum at $(3, 10)$.
3. **(c)** Inside, in reverse: horizontal stretch with scale factor $\frac{1}{2}$ — x -coordinates halve. Maximum at $(\frac{3}{2}, 5)$.

COMMON MISTAKE

The two classic reversals: $f(x + 2)$ moves **left** (not right), and $f(2x)$ **squashes** the graph towards the y -axis (not stretches it away). If in doubt, track one known point through the transformation, like the maximum in the example above.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution — the solutions show full method, not just answers.

Warm-up

Question 1

[3 marks]

The curve $y = (x - 2)(x + 1)(x - 4)$ meets the coordinate axes at four points. Write down the coordinates of all four.

SOLUTION

Each factor gives a root: $x = 2$, $x = -1$ and $x = 4$, so the curve crosses the x -axis at $(2, 0)$, $(-1, 0)$ and $(4, 0)$.

For the y -intercept, substitute $x = 0$:

$$y = (-2)(1)(-4) = 8 \implies (0, 8)$$

Question 2

[3 marks]

Describe the single transformation that maps the graph of $y = f(x)$ onto:

- (a) $y = f(x) + 3$
- (b) $y = f(x - 2)$
- (c) $y = 3f(x)$

SOLUTION

- (a) Outside the function — acts on y : translation 3 **up**.
- (b) Inside the bracket — acts on x , in reverse: translation 2 **right**.
- (c) Outside: **vertical stretch**, scale factor 3.

Question 3

[2 marks]

Write down the equations of the asymptotes of the curve $y = \frac{6}{x}$, and state which pair of quadrants the curve occupies.

SOLUTION

The asymptotes are the axes: $x = 0$ and $y = 0$.

Since $k = 6 > 0$, both coordinates share a sign: the branches sit in the top-right and bottom-left quadrants.

Question 4

[2 marks]

The curve $y = x^2$ is transformed. Write down the equation of the new curve after:

- (a) a translation of 2 to the right,
- (b) a reflection in the x -axis.

SOLUTION

(a) Right means subtracting inside the bracket: $y = (x - 2)^2$.

(b) Reflecting in the x -axis flips y : $y = -x^2$.

Standard**Question 5**

[4 marks]

For the curve $y = (x + 1)(x - 2)^2$:

- (a) Write down where the curve crosses the x -axis and where it touches it.
- (b) Find the y -intercept.
- (c) Describe the behaviour of y as x becomes large and positive.

SOLUTION

(a) The single root $x = -1$ is a crossing; the repeated root $x = 2$ is a touch — the curve meets the axis there and turns back.

(b) Substitute $x = 0$: $y = (1)(-2)^2 = 4$, so $(0, 4)$.

(c) Expanding would start with x^3 , so as $x \rightarrow \infty$, $y \rightarrow \infty$ — the curve climbs away to the top right.

Question 6

[4 marks]

Find the coordinates of the points where the line $y = 2x - 1$ meets the curve $y = x^2 - 4x + 7$.

SOLUTION

1. Set the equations equal:

$$x^2 - 4x + 7 = 2x - 1 \implies x^2 - 6x + 8 = 0$$

2. Factorise: $(x - 2)(x - 4) = 0$, so $x = 2$ or $x = 4$.
3. Use the line for y : $x = 2$ gives $y = 3$; $x = 4$ gives $y = 7$.

The intersection points are $(2, 3)$ and $(4, 7)$.

Question 7

[3 marks]

The curve $y = \frac{k}{x}$ passes through $(2, 5)$.

- (a) Find the value of k .
- (b) Find the y -coordinate of the point on the curve where $x = -4$.

SOLUTION

(a) Substitute the point: $5 = \frac{k}{2}$, so $k = 10$.

(b) $y = \frac{10}{-4} = -\frac{5}{2}$.

Question 8

[3 marks]

The graph of $y = f(x)$ has a maximum point at $(6, -2)$. Write down the coordinates of the corresponding maximum point on:

- (a) $y = f(x - 1)$
- (b) $y = 4f(x)$
- (c) $y = f(3x)$

SOLUTION

(a) Inside the bracket, reversed: translate 1 right — $(7, -2)$.

(b) Vertical stretch $\times 4$ — $(6, -8)$.

(c) Horizontal stretch, scale factor $\frac{1}{3}$ — $(2, -2)$.

Question 9

[4 marks]

Show that the line $y = 3x - 11$ does not intersect the curve $y = x^2 - x - 6$.

SOLUTION

1. An intersection would satisfy both equations:

$$x^2 - x - 6 = 3x - 11 \implies x^2 - 4x + 5 = 0$$

2. Discriminant:

$$\Delta = (-4)^2 - 4(1)(5) = -4 < 0$$

No real solutions, so no intersection points exist. ■

Exam style

Question 10

[5 marks]

$$f(x) = x(x - 2)(x + 3)$$

- (a) Sketch the curve $y = f(x)$, marking where it meets the coordinate axes. [3]
- (b) The curve is translated 2 to the right. Write the equation of the new curve in factorised form, and state its roots. [2]

SOLUTION

(a) Three single roots, so three clean crossings: $x = -3$, $x = 0$, $x = 2$. The y -intercept is at the origin (since $x = 0$ is a root). Positive cubic, so the curve rises from the bottom left, crosses at -3 , turns over a hump, passes through the origin, dips, and crosses again at 2 on its way to the top right.

(b) Translating 2 right replaces x with $x - 2$:

$$y = (x - 2)(x - 4)(x + 1)$$

The roots shift with it: $x = 2$, $x = 4$ and $x = -1$.

Question 11

[4 marks]

The curve $y = x^2$ is mapped onto the curve $y = 2(x + 3)^2 - 5$ by a sequence of transformations.

- (a) Describe a sequence of transformations that achieves this. [3]
- (b) Write down the coordinates of the vertex of the new curve. [1]

SOLUTION

(a) Reading the equation from the inside out:

1. $(x + 3)$ — translate 3 to the **left**;
2. $2(\cdot)$ — **vertical stretch**, scale factor 2;
3. -5 — translate 5 **down**.

(b) The vertex of $y = x^2$ at $(0, 0)$ tracks through: left 3 $\rightarrow (-3, 0)$; the stretch fixes points on the x -axis; down 5 $\rightarrow (-3, -5)$.

Question 12

[4 marks]

The line $y = x + k$, where k is a constant, and the curve $y = \frac{4}{x}$.

Show that the line intersects the curve at two distinct points for **all** values of k .

SOLUTION

1. At an intersection:

$$x + k = \frac{4}{x} \implies x^2 + kx - 4 = 0 \quad (x \neq 0)$$

2. Discriminant:

$$\Delta = k^2 - 4(1)(-4) = k^2 + 16$$

3. $k^2 \geq 0$ for every real k , so $\Delta \geq 16 > 0$ always — two distinct real solutions, and neither can be $x = 0$ (zero doesn't satisfy $x^2 + kx - 4 = 0$). So there are always two intersection points. ■

(Graphically: one crossing on each branch of the reciprocal curve.)