

YEAR 1 PURE · CHAPTER 9

Trigonometry

Specification: Edexcel Pure 5 — Trigonometry · AQA section E

KEY FACTS — THIS CHAPTER

- On the **unit circle**, the point at angle θ is $(\cos \theta, \sin \theta)$ — this defines $\sin$ and $\cos$ for *any* angle.
- **Graphs:** $\sin$ and $\cos$ are waves of period 360° between -1 and 1 ; $\tan$ has period 180° with asymptotes at $90^\circ, 270^\circ, \dots$
- **Exact values** worth memorising: $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 45^\circ = 1$, and the rest of the 30/45/60 family.
- **Identities:** $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$.
- **Solving on an interval:** find the first solution, then use the graph's symmetry to get the rest within range.
- For $\sin(bx)$ or $\cos(bx)$, **widen the interval** for bx first, solve there, then divide back.

In a right-angled triangle, $\sin$, $\cos$ and $\tan$ only make sense for angles between 0° and 90° . The unit circle frees them — giving every angle a sine and cosine, and turning trig into the wave functions that model anything that repeats.

1. The unit circle

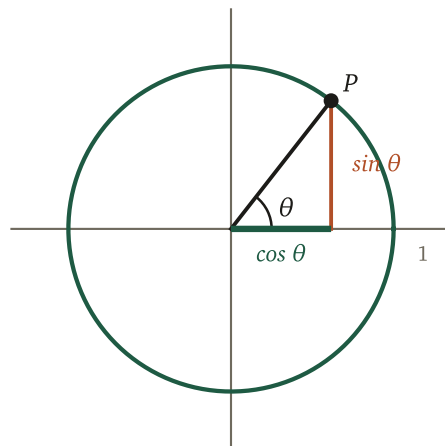
Place a circle of radius 1 at the origin and mark the point reached by rotating θ from the positive x -axis.

KEY RESULT

The point on the unit circle at angle θ has coordinates

$$(\cos \theta, \sin \theta).$$

So $\cos \theta$ is its **horizontal** position and $\sin \theta$ its **vertical** position — for any θ , not just acute ones. This is why $\sin$ and $\cos$ can be negative: past 90° the point moves into other quadrants.



$P = (\cos \theta, \sin \theta)$ on the unit circle

$\cos \theta$ across, $\sin \theta$ up — the coordinates of the point at angle θ .

Exact values

A few angles have exact sin/cos/tan worth knowing by heart — they come from the 30–60–90 and 45–45–90 triangles and appear constantly in exam answers.

THE VALUES TO MEMORISE

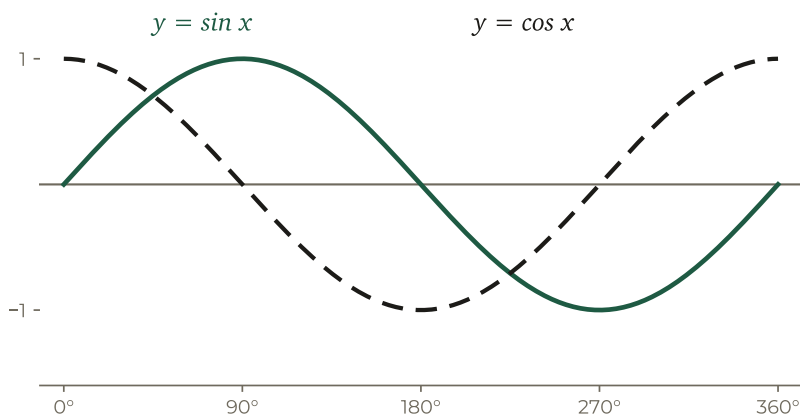
θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—

2. The graphs

Let θ run all the way round the circle (and beyond) and the height $\sin \theta$ and width $\cos \theta$ trace out waves.

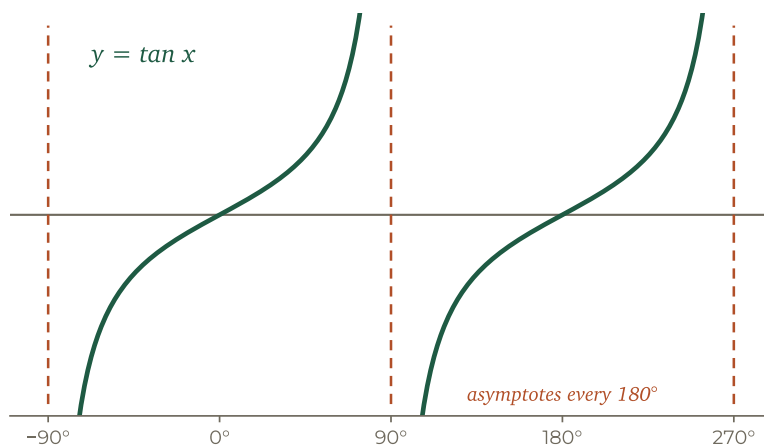
KEY RESULT

- $y = \sin x$ and $y = \cos x$ are waves of **period** 360° , oscillating between -1 and 1 . The cosine graph is the sine graph shifted left by 90° .
- $y = \tan x$ has **period** 180° and shoots off to $\pm\infty$ at $x = 90^\circ, 270^\circ, \dots$ — vertical **asymptotes** where $\cos x = 0$.



both repeat every 360° and run between -1 and 1

sin and cos: same wave, a 90° shift apart.



tan repeats every 180° , with asymptotes wherever $\cos x = 0$.

These shapes are what make solving equations possible: once you know *one* solution, the graph's symmetry hands you all the others.

3. Trig identities

Two identities connect the three functions, and both fall straight out of the unit circle.

KEY RESULT

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \sin^2 \theta + \cos^2 \theta = 1$$

WHY IT WORKS

On the unit circle the point $(\cos \theta, \sin \theta)$ is exactly 1 from the origin, so Pythagoras gives $\cos^2 \theta + \sin^2 \theta = 1$ immediately. And in the right-angled triangle formed by the radius, $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$.

WORKED EXAMPLE 9.1 · USING THE IDENTITY

Given that $\sin \theta = \frac{3}{5}$ and θ is acute, find the exact values of $\cos \theta$ and $\tan \theta$.

1. Use $\sin^2 \theta + \cos^2 \theta = 1$:

$$\cos^2 \theta = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

2. θ is acute, so $\cos \theta$ is positive:

$$\cos \theta = \frac{4}{5}$$

3. Then

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

COMMON MISTAKE

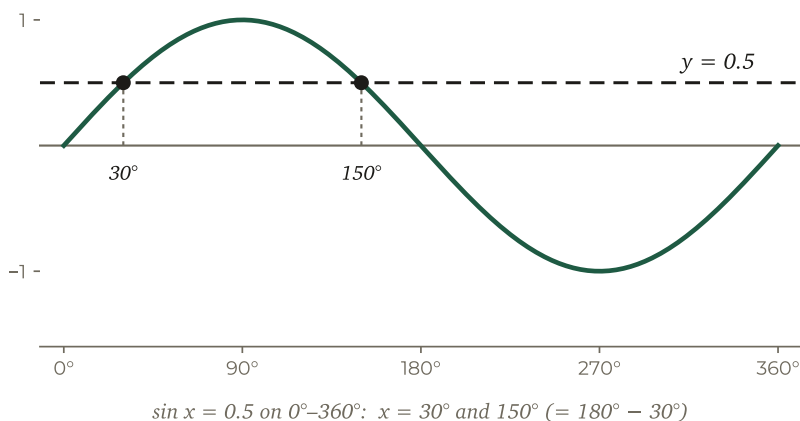
When you square-root to find $\cos \theta$, the sign depends on the quadrant. For an **acute** angle cosine is positive, but for an obtuse θ it would be negative — always check which quadrant θ lives in before fixing the sign.

4. Solving trig equations

To solve something like $\sin x = c$ on a given interval, find the first solution with your calculator, then read the rest off the graph using its symmetry.

THE METHOD

1. Get the **principal value** from $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.
2. Use the graph's symmetry for the others in range: - sin: a second solution at $180^\circ - x$; - cos: a second solution at $360^\circ - x$; - tan: add 180° each time.
3. Add or subtract 360° to land any further solutions inside the interval.



The line $y = 0.5$ meets the sine curve twice in 0° – 360° : at 30° and $180^\circ - 30^\circ$.

WORKED EXAMPLE 9.2 • A COSINE EQUATION

Solve $\cos x = -\frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$.

1. Principal value: $\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$, so the related acute angle is 60° . Since cosine is **negative**, x is in the second and third quadrants.
2. Second-quadrant solution: $x = 180^\circ - 60^\circ = 120^\circ$. Third-quadrant solution: $x = 180^\circ + 60^\circ = 240^\circ$.

So $x = 120^\circ$ or $x = 240^\circ$.

WORKED EXAMPLE 9.3 · A MULTIPLE ANGLE — WIDEN THE INTERVAL

Solve $\sin 2x = \frac{\sqrt{3}}{2}$ for $0^\circ \leq x \leq 360^\circ$.

1. Let $u = 2x$. As x runs over 0° to 360° , u runs over 0° to 720° — solve in this **wider** interval.
2. $\sin u = \frac{\sqrt{3}}{2}$ has principal value $u = 60^\circ$, with sine positive in quadrants 1 and 2:

$$u = 60^\circ, 120^\circ, 60^\circ + 360^\circ, 120^\circ + 360^\circ = 60^\circ, 120^\circ, 420^\circ, 480^\circ$$

3. Divide each by 2 to get x :

$$x = 30^\circ, 60^\circ, 210^\circ, 240^\circ$$

COMMON MISTAKE

With $\sin 2x$, don't solve for x and *then* worry about extra solutions — you'll miss half of them. Widen the interval to cover $2x$ **first**, collect every solution there, and divide back at the end.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution — the solutions show full method, not just answers. Angles are in degrees.

Warm-up**Question 1**

[3 marks]

Write down the exact values of:

- (a) $\sin 60^\circ$
- (b) $\cos 45^\circ$
- (c) $\tan 30^\circ$

SOLUTION

$$(a) \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

$$(b) \cos 45^\circ = \frac{\sqrt{2}}{2} \text{ (also written } \frac{1}{\sqrt{2}}).$$

$$(c) \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

Question 2

[3 marks]

Solve $\sin x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

Principal value $\sin^{-1}(\frac{1}{2}) = 30^\circ$. Sine is positive in quadrants 1 and 2, so the second solution is $180^\circ - 30^\circ = 150^\circ$.

$$x = 30^\circ \text{ or } 150^\circ$$

Question 3

[2 marks]

State the period of each graph: (a) $y = \sin x$, (b) $y = \tan x$.

SOLUTION

(a) 360° . (b) 180° .

Question 4

[3 marks]

Given that $\cos \theta = \frac{5}{13}$ and θ is acute, find $\sin \theta$.

SOLUTION

Using $\sin^2 \theta + \cos^2 \theta = 1$:

$$\sin^2 \theta = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

θ is acute, so $\sin \theta = \frac{12}{13}$.

Standard**Question 5**

[3 marks]

Solve $\cos x = -\frac{\sqrt{3}}{2}$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

The related acute angle is $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$. Cosine is negative in quadrants 2 and 3:

$$x = 180^\circ - 30^\circ = 150^\circ \quad \text{or} \quad x = 180^\circ + 30^\circ = 210^\circ$$

Question 6

[4 marks]

Solve $\tan x = \sqrt{3}$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

Principal value $\tan^{-1}(\sqrt{3}) = 60^\circ$. Tangent has period 180° , so add 180° for the next solution in range:

$$x = 60^\circ \quad \text{or} \quad x = 60^\circ + 180^\circ = 240^\circ$$

Question 7

[4 marks]

Given that $\sin \theta = \frac{2}{3}$ and θ is obtuse, find the exact values of $\cos \theta$ and $\tan \theta$.

SOLUTION

$$1. \cos^2 \theta = 1 - \left(\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9}.$$

2. θ is **obtuse** (second quadrant), so $\cos \theta$ is negative:

$$\cos \theta = -\frac{\sqrt{5}}{3}$$

$$3. \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{2/3}{-\sqrt{5}/3} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}.$$

Question 8

[5 marks]

Solve $2 \sin x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$, then state how many solutions $2 \sin x - 1 = 0$ has for $0^\circ \leq x \leq 720^\circ$.

SOLUTION

1. Rearrange: $\sin x = \frac{1}{2}$.
2. On 0° – 360° : $x = 30^\circ$ or 150° (as in Question 2).
3. Over 0° – 720° the sine graph repeats once more, adding $30^\circ + 360^\circ = 390^\circ$ and $150^\circ + 360^\circ = 510^\circ$ — so 4 solutions in total.

Question 9

[5 marks]

Solve $\cos 2x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

1. Let $u = 2x$; then $0^\circ \leq u \leq 720^\circ$.
2. $\cos u = \frac{1}{2}$ has acute angle 60° ; cosine is positive in quadrants 1 and 4, so in 0° – 360° , $u = 60^\circ$ or 300° . Adding 360° : $u = 420^\circ$ or 660° as well.
3. Divide by 2:

$$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

Exam style**Question 10**

[5 marks]

Solve $\sin x = \cos x$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

Divide both sides by $\cos x$ (valid where $\cos x \neq 0$):

$$\frac{\sin x}{\cos x} = 1 \implies \tan x = 1$$

Principal value 45° ; tangent period 180° :

$$x = 45^\circ \quad \text{or} \quad x = 225^\circ$$

Question 11

[6 marks]

Solve $2 \sin^2 x + \sin x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

1. This is a quadratic in $\sin x$. Let $s = \sin x$: $2s^2 + s - 1 = 0$.
2. Factorise: $(2s - 1)(s + 1) = 0$, so $s = \frac{1}{2}$ or $s = -1$.
3. Solve each on 0° – 360° :
 $-\sin x = \frac{1}{2}$: $x = 30^\circ$ or 150° . $-\sin x = -1$: $x = 270^\circ$.

So $x = 30^\circ, 150^\circ, 270^\circ$.

Question 12

[6 marks]

Solve $\sin(x + 30^\circ) = \frac{\sqrt{3}}{2}$ for $0^\circ \leq x \leq 360^\circ$.

SOLUTION

1. Let $u = x + 30^\circ$. As x runs 0° – 360° , u runs 30° – 390° .
2. $\sin u = \frac{\sqrt{3}}{2}$ has acute angle 60° ; sine positive in quadrants 1 and 2. In range 30° – 390° :

$$u = 60^\circ, 120^\circ, 60^\circ + 360^\circ = 420^\circ \text{ (too big)}$$

so $u = 60^\circ$ or 120° (note $u = 420^\circ$ exceeds 390°).

3. Subtract 30° :

$$x = 30^\circ \quad \text{or} \quad x = 90^\circ$$