

YEAR 1 PURE · CHAPTER 4

Coordinate Geometry of Straight Lines

Specification: Edexcel Pure 3 — Coordinate geometry · AQA section C

KEY FACTS — THIS CHAPTER

- Gradient between two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$ — rise over run.
- Equation of a line: $y = mx + c$ (gradient m , y -intercept c), or $y - y_1 = m(x - x_1)$ when you know a point on it.
- **Parallel** lines have **equal** gradients; **perpendicular** lines have gradients multiplying to -1 : $m_2 = -\frac{1}{m_1}$.
- Midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ — average the coordinates.
- Distance: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Pythagoras in disguise.

Straight lines are the workhorses of coordinate geometry. Master the handful of tools in this chapter — gradient, equation, midpoint, distance, perpendicularity — and you’ve already done most of the work for circles (Chapter 5) and tangents and normals in differentiation later on.

1. Gradient and the equation of a line

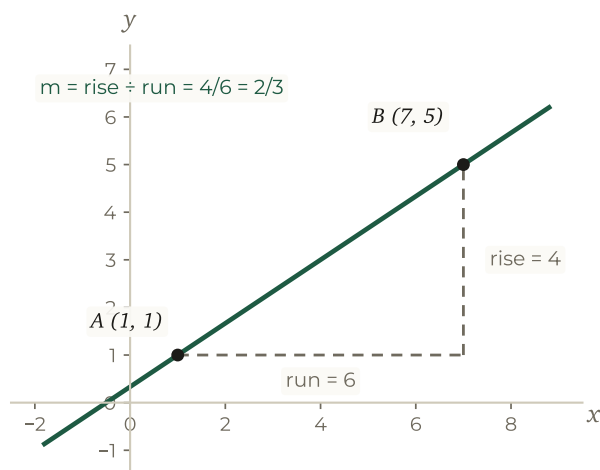
The **gradient** measures steepness: how much the line climbs for every unit it moves right.

KEY RESULT

Between any two points (x_1, y_1) and (x_2, y_2) on the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

A negative gradient falls from left to right; a horizontal line has $m = 0$; a vertical line has no gradient (its equation is $x = a$).



Any two points on the line give the same rise ÷ run — that's what makes it straight.

TWO WAYS TO WRITE A LINE

- $y = mx + c$ — perfect when you know the gradient and the y -intercept.
- $y - y_1 = m(x - x_1)$ — perfect when you know the gradient and **any** point (x_1, y_1) ; no need to find c first.

Exam answers sometimes ask for the form $ax + by + c = 0$ with integer coefficients — multiply through to clear fractions and gather everything on one side.

WORKED EXAMPLE 4.1 · A LINE THROUGH TWO POINTS

Find the equation of the line through $(-1, 3)$ and $(2, -3)$.

1. Gradient first:

$$m = \frac{-3 - 3}{2 - (-1)} = \frac{-6}{3} = -2$$

2. Use $y - y_1 = m(x - x_1)$ with either point — take $(-1, 3)$:

$$y - 3 = -2(x + 1)$$

3. Tidy:

$$y = -2x + 1$$

(Check with the other point: $x = 2$ gives $y = -3$. ✓)

COMMON MISTAKE

Keep the subtraction order consistent: $\frac{y_2 - y_1}{x_2 - x_1}$, both differences the same way round. Swapping one but not the other flips the sign of the gradient — the single most common slip with lines.

2. Midpoints and distance

Two more tools, both straight from a picture.

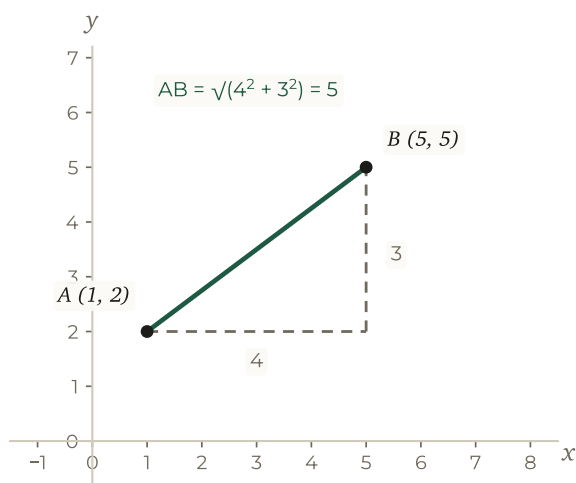
KEY RESULT

For $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The midpoint just **averages** the coordinates. The distance is Pythagoras: the horizontal and vertical gaps are the two legs, and AB is the hypotenuse.



Distance is Pythagoras: legs 4 and 3, hypotenuse 5.

WORKED EXAMPLE 4.2 · MIDPOINT AND LENGTH OF A SEGMENT

For $A(-3, 2)$ and $B(5, 8)$, find the midpoint of AB and the exact length AB .

1. Average the coordinates:

$$\text{midpoint} = \left(\frac{-3+5}{2}, \frac{2+8}{2} \right) = (1, 5)$$

2. Pythagoras on the gaps (8 across, 6 up):

$$AB = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

3. Parallel and perpendicular lines**KEY RESULT**

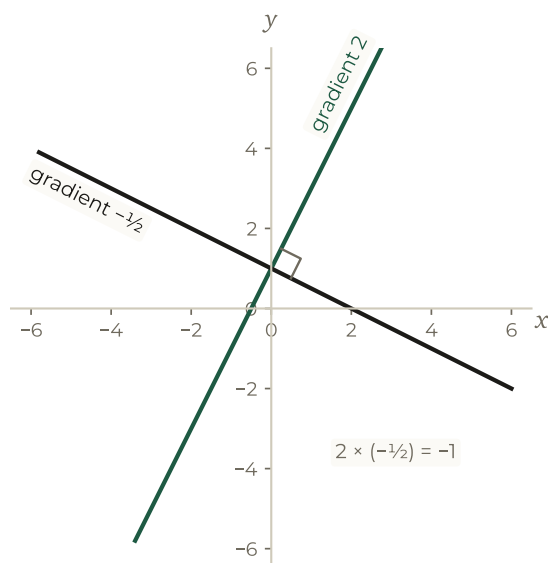
- **Parallel** lines never meet because they climb at the same rate: $m_1 = m_2$.
- **Perpendicular** lines meet at right angles, and their gradients multiply to -1 :

$$m_1 m_2 = -1 \quad \Longleftrightarrow \quad m_2 = -\frac{1}{m_1}$$

— flip the fraction, flip the sign. The perpendicular to gradient $\frac{2}{3}$ has gradient $-\frac{3}{2}$.

WHY IT WORKS

Rotating a line a quarter turn swaps its rise and run — what used to be “along” is now “up” — and sends the line uphill if it was downhill. So a gradient $\frac{a}{b}$ becomes $-\frac{b}{a}$, and the product is $\frac{a}{b} \times \left(-\frac{b}{a}\right) = -1$.



Gradients 2 and $-\frac{1}{2}$: flip the fraction, flip the sign, product -1 .

WORKED EXAMPLE 4.3 · A PARALLEL LINE THROUGH A POINT

Find the equation of the line through $(2, -1)$ that is parallel to the line $3x + y = 7$.

1. Rearrange the given line to read off its gradient:

$$y = -3x + 7 \implies m = -3$$

2. Parallel means the **same** gradient, so build the line through $(2, -1)$ with $m = -3$:

$$y - (-1) = -3(x - 2) \implies y = -3x + 5$$

WORKED EXAMPLE 4.4 · A PERPENDICULAR LINE THROUGH A POINT

Find the equation of the line through $(6, 1)$ that is perpendicular to $y = 3x + 2$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

1. The given gradient is 3, so the perpendicular gradient is $-\frac{1}{3}$ — flip and negate.
2. Line through $(6, 1)$:

$$y - 1 = -\frac{1}{3}(x - 6)$$

3. Multiply through by 3 to clear the fraction, and gather:

$$3y - 3 = -(x - 6) \implies x + 3y - 9 = 0$$

COMMON MISTAKE

Perpendicular needs **both** moves — flip the fraction *and* flip the sign. The perpendicular to $m = 3$ is $-\frac{1}{3}$, not $\frac{1}{3}$ (sign forgotten) and not -3 (flip forgotten).

4. Modelling with straight lines

A linear model is just $y = mx + c$ wearing real-world clothes — and exam questions love asking what m and c *mean* in context.

KEY RESULT

In a linear model $y = mx + c$:

- m is the **rate of change** — how much y changes for each unit of x (cost per mile, depth per minute, ...);
- c is the **starting value** — what y is when $x = 0$ (the fixed charge, the initial depth, ...).

WORKED EXAMPLE 4.5 · BUILDING AND READING A LINEAR MODEL

A taxi journey costs £6.80 for 2 miles and £12.20 for 5 miles. Assuming cost C (£) is linear in distance d (miles), find the model and interpret its constants.

1. The rate per mile is the gradient:

$$m = \frac{12.20 - 6.80}{5 - 2} = \frac{5.40}{3} = 1.80$$

2. Use a known point to find the constant: $6.80 = 1.80(2) + c$ gives $c = 3.20$.

$$C = 1.8d + 3.2$$

3. Interpret: each extra mile costs **£1.80** (m), and the fixed charge just for getting in the taxi is **£3.20** (c).

Practice

Twelve questions in three tiers. Try each one properly before opening the solution — the solutions show full method, not just answers.

Warm-up

Question 1

[3 marks]

Find the equation of the line through $(1, 2)$ and $(4, 11)$, giving your answer in the form $y = mx + c$.

SOLUTION

1. Gradient:

$$m = \frac{11 - 2}{4 - 1} = \frac{9}{3} = 3$$

2. Line through $(1, 2)$ with gradient 3:

$$y - 2 = 3(x - 1) \implies y = 3x - 1$$

(Check: $x = 4$ gives $y = 11$. ✓)

Question 2

[3 marks]

For the points $A(-2, 3)$ and $B(4, 11)$, find:

- (a) the midpoint of AB ,
- (b) the length AB .

SOLUTION

- (a) Average the coordinates:

$$\left(\frac{-2 + 4}{2}, \frac{3 + 11}{2} \right) = (1, 7)$$

(b) Pythagoras on the gaps (6 across, 8 up):

$$AB = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

Question 3

[2 marks]

Write down the gradient and y -intercept of the line $3x + 4y - 8 = 0$.

SOLUTION

Rearrange for y :

$$4y = -3x + 8 \implies y = -\frac{3}{4}x + 2$$

Gradient $-\frac{3}{4}$, y -intercept $(0, 2)$.

Question 4

[2 marks]

(a) Show that the lines $y = 2x - 1$ and $2y - 4x + 5 = 0$ are parallel.

(b) Write down the gradient of any line perpendicular to them.

SOLUTION

(a) Rearranging the second: $2y = 4x - 5$, so $y = 2x - \frac{5}{2}$. Both have gradient 2 — parallel. (Different intercepts, so they're distinct lines, not the same one.)

(b) Flip and negate: $-\frac{1}{2}$.

Standard

Question 5

[3 marks]

Find the equation of the line through $(3, -1)$ that is parallel to the line $2x + 3y = 6$.

SOLUTION

1. The given line rearranges to $y = -\frac{2}{3}x + 2$, so $m = -\frac{2}{3}$.

2. Same gradient through $(3, -1)$:

$$y + 1 = -\frac{2}{3}(x - 3) \implies y = -\frac{2}{3}x + 1$$

Question 6

[3 marks]

Find the equation of the line through $(4, 5)$ that is perpendicular to $y = 2x - 3$.

SOLUTION

1. Perpendicular gradient: flip and negate 2 to get $-\frac{1}{2}$.
2. Through $(4, 5)$:

$$y - 5 = -\frac{1}{2}(x - 4) \implies y = -\frac{1}{2}x + 7$$

Question 7

[4 marks]

The points $A(2, 3)$, $B(6, 5)$ and $C(4, 9)$ form a triangle. Show that the triangle is right-angled and isosceles.

SOLUTION

1. Gradients of the sides:

$$m_{AB} = \frac{5 - 3}{6 - 2} = \frac{1}{2} \quad m_{BC} = \frac{9 - 5}{4 - 6} = -2$$

2. $m_{AB} \times m_{BC} = \frac{1}{2} \times (-2) = -1$, so $AB \perp BC$ — the triangle is right-angled at B . ✓
3. Lengths of those two sides:

$$AB = \sqrt{4^2 + 2^2} = \sqrt{20} \quad BC = \sqrt{2^2 + 4^2} = \sqrt{20}$$

Two equal sides, so the triangle is also isosceles. ✓

Question 8

[4 marks]

Find the equation of the perpendicular bisector of the points $A(1, 4)$ and $B(7, 2)$.

SOLUTION

1. The bisector passes through the midpoint:

$$\left(\frac{1+7}{2}, \frac{4+2}{2} \right) = (4, 3)$$

2. Gradient of AB : $\frac{2-4}{7-1} = -\frac{1}{3}$, so the perpendicular gradient is 3.

3. Through $(4, 3)$ with gradient 3:

$$y - 3 = 3(x - 4) \implies y = 3x - 9$$

(This is the Chapter 5 workhorse: a circle's centre always sits on the perpendicular bisector of any chord.)

Question 9

[3 marks]

The line through $P(2, 5)$ and $Q(6, k)$ is perpendicular to the line $y = -2x + 1$. Find the value of k .

SOLUTION

1. Perpendicular to gradient -2 means gradient $\frac{1}{2}$.
2. Set the gradient of PQ equal to it:

$$\frac{k - 5}{6 - 2} = \frac{1}{2}$$

3. Solve: $k - 5 = 2$, so $k = 7$.

Exam style

Question 10

[6 marks]

The line l_1 has equation $y = 2x - 3$. The line l_2 is perpendicular to l_1 and passes through the point $(8, 3)$.

- (a) Find the equation of l_2 . [3]
- (b) Find the coordinates of the point where l_1 and l_2 intersect. [3]

SOLUTION

(a)

1. Perpendicular gradient to 2: $-\frac{1}{2}$.
2. Through $(8, 3)$:

$$y - 3 = -\frac{1}{2}(x - 8) \implies y = -\frac{1}{2}x + 7$$

(b)

1. Set the two equations equal:

$$2x - 3 = -\frac{1}{2}x + 7$$

2. Multiply by 2 and solve: $4x - 6 = -x + 14$, so $5x = 20$ and $x = 4$.
3. Substitute back: $y = 2(4) - 3 = 5$. The lines meet at $(4, 5)$.

Question 11

[5 marks]

The points $A(1, 2)$, $B(5, 4)$, $C(7, 8)$ and $D(3, 6)$ are the vertices of a quadrilateral. Show that $ABCD$ is a rhombus but **not** a square.

SOLUTION

1. Opposite sides parallel (gradients):

$$m_{AB} = \frac{4-2}{5-1} = \frac{1}{2} = m_{DC} \quad m_{BC} = \frac{8-4}{7-5} = 2 = m_{AD}$$

so $ABCD$ is a parallelogram.

2. Adjacent sides equal:

$$AB = \sqrt{4^2 + 2^2} = \sqrt{20} \quad BC = \sqrt{2^2 + 4^2} = \sqrt{20}$$

A parallelogram with equal adjacent sides has all four sides equal — a **rhombus**. ✓

3. Not a square: adjacent sides would have to be perpendicular, but

$$m_{AB} \times m_{BC} = \frac{1}{2} \times 2 = 1 \neq -1$$

so the corners aren't right angles. ✓

Question 12

[5 marks]

A water tank is being drained. After 2 minutes the depth of water is 46 cm; after 7 minutes it is 31 cm. Assume the depth d cm after t minutes follows a linear model.

- (a) Find the model in the form $d = mt + c$. [3]
- (b) Interpret the values of m and c in context. [1]
- (c) Find how long it takes for the tank to empty completely. [1]

SOLUTION**(a)**

1. Gradient:

$$m = \frac{31 - 46}{7 - 2} = \frac{-15}{5} = -3$$

2. Use $(2, 46)$: $46 = -3(2) + c$, so $c = 52$.

$$d = -3t + 52$$

(b) The depth falls by 3 cm every minute ($m = -3$); the tank started at 52 cm deep ($c = 52$, the depth at $t = 0$).

(c) Empty means $d = 0$:

$$0 = -3t + 52 \implies t = \frac{52}{3} \approx 17.3 \text{ minutes}$$