

YEAR 1 PURE • CHAPTER 1

Indices and Surds

Specification: Edexcel Pure 2 — Algebra and functions • AQA section B

KEY FACTS — THIS CHAPTER

- $a^m \times a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$.
- $a^0 = 1$ (for $a \neq 0$), $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ — **root first, then power**.
- $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ — simplify surds by pulling out the **largest square factor**.
- To rationalise $\frac{k}{\sqrt{b}}$, multiply top and bottom by $\sqrt{b}$.
- To rationalise $\frac{k}{a + \sqrt{b}}$, multiply top and bottom by the **conjugate** $a - \sqrt{b}$, because $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

Indices and surds are the grammar of A-level algebra — every later chapter assumes you can move between powers, roots and fractions without breaking stride. The good news: there are only a handful of rules, and every one of them exists for a reason you can see.

1. The laws of indices

A power is just a count of repeated factors: 2^3 means $2 \times 2 \times 2$. Keep that picture in mind and the laws stop being things to memorise — they become things you could reinvent on the spot.

THE INDEX LAWS

For any base a and indices m, n :

$$a^m \times a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad (a^m)^n = a^{mn}$$

The first two need the **same base**; you can't combine $2^3 \times 5^2$ this way.

WHY IT WORKS

Multiplying $a^3 \times a^4$ just lines up 3 copies of a next to 4 more — seven copies in total, so a^{3+4} . Dividing cancels copies pair by pair, which subtracts the counts. And $(a^3)^4$ is four groups of three copies: 3×4 copies altogether.

WORKED EXAMPLE 1.1 • SIMPLIFYING WITH THE INDEX LAWS

Simplify $(2x^2)^3 \times 3x^{-4}$.

1. Cube the whole bracket — the 2 as well as the x^2 :

$$(2x^2)^3 = 2^3 \times x^{2 \times 3} = 8x^6$$

2. Multiply the coefficients and add the indices:

$$8x^6 \times 3x^{-4} = 24x^{6+(-4)} = 24x^2$$

COMMON MISTAKE

$(2x^2)^3$ is $8x^6$, not $2x^6$ — the power applies to **everything** inside the bracket, coefficient included. It's the single most common indices slip at A-level.

2. Zero, negative and fractional indices

What should a^0 or $a^{\frac{1}{2}}$ mean? You can't repeat a factor "half a time" — instead, we choose the meanings that keep the index laws working, and those choices turn out to be forced.

KEY RESULT

$$a^0 = 1 \quad (a \neq 0) \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

WHY IT WORKS

The laws leave no other option. Since $a^n \times a^{-n} = a^{n-n} = a^0$, and dividing anything by itself gives 1, we need $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$. Since $\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2} \times 2} = a$, the number $a^{\frac{1}{2}}$ squares to a — it is $\sqrt{a}$.

WORKED EXAMPLE 1.2 • EVALUATING NEGATIVE AND FRACTIONAL INDICES

Evaluate $8^{\frac{2}{3}}$, $25^{-\frac{1}{2}}$ and $\left(\frac{16}{9}\right)^{-\frac{1}{2}}$.

1. For $8^{\frac{2}{3}}$, take the root first, then the power:

$$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

2. For $25^{-\frac{1}{2}}$, the minus sign means “one over”:

$$25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{5}$$

3. For a fraction, a negative power flips it first:

$$\left(\frac{16}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

COMMON MISTAKE

Root first, then power — $\left(\sqrt[3]{8}\right)^2 = 4$ is far friendlier than $\sqrt[3]{64}$, even though both work. And a negative index makes a **reciprocal**, never a negative number: $25^{-\frac{1}{2}}$ is $\frac{1}{5}$, not -5 .

3. Surds

A **surd** is a root that doesn’t simplify to a whole number or fraction — $\sqrt{2}$, $\sqrt{3}$, $5\sqrt{2}$. We leave them as they are because they’re *exact*: writing 1.414 throws information away, and exam mark schemes notice.

KEY RESULT

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

To simplify a surd, hunt for the **largest square factor** hiding inside it.

WORKED EXAMPLE 1.3 • SIMPLIFYING SURDS

Simplify $\sqrt{75} + \sqrt{27}$.

1. Pull the largest square factor out of each:

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3} \qquad \sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

2. Both are now multiples of the same surd, so they add like like-terms:

$$5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3}$$

COMMON MISTAKE

The rules split **products**, never sums: $\sqrt{9 \times 16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 7$. There is no rule for $\sqrt{a + b}$.

4. Rationalising the denominator

Answers are easier to compare, combine and mark when no surd sits underneath a fraction — so we **rationalise**: rebuild the fraction with a whole-number denominator without changing its value.

CASE 1 — A SINGLE SURD UNDERNEATH

Multiply top and bottom by that surd:

$$\frac{k}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{k\sqrt{b}}{b}$$

Multiplying by $\frac{\sqrt{b}}{\sqrt{b}} = 1$ leaves the value untouched — it only changes how the fraction is written.

WORKED EXAMPLE 1.4 • RATIONALISING A SINGLE SURD

Rationalise $\frac{14}{\sqrt{7}}$.

1. Multiply top and bottom by $\sqrt{7}$:

$$\frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{7}}{7}$$

2. Simplify the fraction: $\frac{14\sqrt{7}}{7} = 2\sqrt{7}$.

CASE 2 — TWO TERMS UNDERNEATH: USE THE CONJUGATE

For a denominator like $a + \sqrt{b}$, multiply top and bottom by its **conjugate** $a - \sqrt{b}$ (same terms, opposite sign on the surd):

$$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

WHY IT WORKS

It's the difference of two squares. Expanding gives $a^2 - a\sqrt{b} + a\sqrt{b} - b$: the two surd terms are equal and opposite, so they cancel each other exactly, leaving the rational number $a^2 - b$. The conjugate is designed to make that cancellation happen.

WORKED EXAMPLE 1.5 • RATIONALISING WITH A CONJUGATE

Rationalise $\frac{6}{\sqrt{7} + 1}$.

1. The conjugate of $\sqrt{7} + 1$ is $\sqrt{7} - 1$. Multiply top and bottom by it:

$$\frac{6}{\sqrt{7} + 1} \times \frac{\sqrt{7} - 1}{\sqrt{7} - 1} = \frac{6(\sqrt{7} - 1)}{(\sqrt{7})^2 - 1^2}$$

2. The denominator collapses to a whole number:

$$(\sqrt{7})^2 - 1^2 = 7 - 1 = 6$$

3. Cancel the 6:

$$\frac{6(\sqrt{7} - 1)}{6} = \sqrt{7} - 1$$

WORKED EXAMPLE 1.6 • A HARDER CONJUGATE

Write $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

1. Multiply top and bottom by the conjugate $2 + \sqrt{3}$:

$$\frac{2 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{(2 + \sqrt{3})^2}{2^2 - 3}$$

2. Expand the top and simplify the bottom:

$$(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3} \qquad 2^2 - 3 = 1$$

3. Dividing by 1 changes nothing:

$$\frac{2 + \sqrt{3}}{2 - \sqrt{3}} = 7 + 4\sqrt{3}$$

So $a = 7$ and $b = 4$.

COMMON MISTAKE

Multiply the **top and bottom** — rationalising is multiplying by a clever form of 1, and forgetting the numerator changes the value. And the conjugate flips only the sign of the **surd** term: the conjugate of $4 - \sqrt{2}$ is $4 + \sqrt{2}$, not $-4 + \sqrt{2}$.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution — the solutions show full method, not just answers.

Warm-up**Question 1**

[3 marks]

Write each of the following as a single power:

(a) $2^3 \times 2^4$

(b) $\frac{3^5}{3^2}$

(c) $(5^2)^3$

SOLUTION

(a) Same base, multiplying — add the indices: $2^3 \times 2^4 = 2^{3+4} = 2^7$.

(b) Same base, dividing — subtract: $\frac{3^5}{3^2} = 3^{5-2} = 3^3$.

(c) Power of a power — multiply: $(5^2)^3 = 5^{2 \times 3} = 5^6$.

Question 2

[3 marks]

Evaluate:

(a) 7^0

(b) 4^{-2}

(c) $27^{\frac{1}{3}}$

SOLUTION

(a) Anything (non-zero) to the power 0 is 1: $7^0 = 1$.

(b) Negative index means reciprocal: $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$.

(c) A $\frac{1}{3}$ power is a cube root: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$.

Question 3

[3 marks]

Simplify:

(a) $\sqrt{48}$

(b) $\sqrt{18} + \sqrt{8}$

SOLUTION

(a) The largest square factor of 48 is 16:

$$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

(b) Simplify each surd, then add like terms:

$$\sqrt{18} + \sqrt{8} = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$$

Question 4

[2 marks]

Simplify:

(a) $\sqrt{3} \times \sqrt{12}$

(b) $\frac{\sqrt{40}}{\sqrt{10}}$

SOLUTION

(a) Multiply under one root: $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$.

(b) Divide under one root: $\frac{\sqrt{40}}{\sqrt{10}} = \sqrt{4} = 2$.

Standard**Question 5**

[3 marks]

Simplify $(2x^2y^3)^2 \times 3x^{-4}y$, giving your answer without negative indices.

SOLUTION

1. Square the whole bracket — coefficient included:

$$(2x^2y^3)^2 = 4x^4y^6$$

2. Multiply coefficients and add the indices of each letter:

$$4x^4y^6 \times 3x^{-4}y = 12x^{4+(-4)}y^{6+1} = 12x^0y^7 = 12y^7$$

Question 6

[4 marks]

Evaluate:

(a) $8^{\frac{2}{3}}$

(b) $32^{-\frac{3}{5}}$

(c) $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$

SOLUTION(a) Root first, then power: $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$.

(b) Root, power, then reciprocal:

$$32^{-\frac{3}{5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = \frac{1}{8}$$

(c) The negative power flips the fraction first:

$$\left(\frac{9}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{2}{3}$$

Question 7

[3 marks]

Expand and simplify $(5 - \sqrt{3})(2 + \sqrt{3})$.**SOLUTION**

Expand all four terms:

$$5 \times 2 + 5\sqrt{3} - 2\sqrt{3} - (\sqrt{3})^2 = 10 + 5\sqrt{3} - 2\sqrt{3} - 3$$

Collect the rational and surd parts separately:

$$(10 - 3) + (5 - 2)\sqrt{3} = 7 + 3\sqrt{3}$$

Question 8

[3 marks]

Rationalise the denominator of each fraction, simplifying fully:

(a) $\frac{6}{\sqrt{3}}$

(b) $\frac{10}{\sqrt{5}}$

SOLUTION

(a) Multiply top and bottom by $\sqrt{3}$:

$$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

(b) Multiply top and bottom by $\sqrt{5}$:

$$\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

Question 9

[3 marks]

Solve $2^{x+1} = 8^{x-1}$.

SOLUTION

1. Write both sides with the same base — $8 = 2^3$:

$$2^{x+1} = (2^3)^{x-1} = 2^{3x-3}$$

2. Same base on both sides, so the indices must be equal:

$$x + 1 = 3x - 3$$

3. Solve: $4 = 2x$, so $x = 2$.

(Check: $2^3 = 8$ and $8^1 = 8$. ✓)

Exam style**Question 10**

[5 marks]

Rationalise the denominator of each fraction, giving each answer in its simplest form:

(a) $\frac{5}{\sqrt{7}-2}$ [2]

(b) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ [3]

SOLUTION

(a) Multiply top and bottom by the conjugate $\sqrt{7}+2$:

$$\frac{5}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{5(\sqrt{7}+2)}{7-4} = \frac{5\sqrt{7}+10}{3}$$

(b) Multiply top and bottom by the conjugate $3+\sqrt{2}$:

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{(3+\sqrt{2})^2}{9-2}$$

Expanding the top: $(3+\sqrt{2})^2 = 9+6\sqrt{2}+2 = 11+6\sqrt{2}$, so the answer is

$$\frac{11+6\sqrt{2}}{7}$$

Question 11

[5 marks]

(a) Write $(2+\sqrt{5})^2$ in the form $a+b\sqrt{5}$, where a and b are integers. [2]

(b) Hence, or otherwise, show that $\frac{1}{(2+\sqrt{5})^2} = 9-4\sqrt{5}$. [3]

SOLUTION

(a) Expand:

$$(2+\sqrt{5})^2 = 4+4\sqrt{5}+5 = 9+4\sqrt{5}$$

(b) Using part (a), rationalise with the conjugate $9 - 4\sqrt{5}$:

$$\frac{1}{9 + 4\sqrt{5}} \times \frac{9 - 4\sqrt{5}}{9 - 4\sqrt{5}} = \frac{9 - 4\sqrt{5}}{81 - 80} = 9 - 4\sqrt{5} \blacksquare$$

(The denominator $81 - 16 \times 5 = 1$ is what makes this one tidy.)

Question 12

[4 marks]

A rectangle has width $(5 - \sqrt{5})$ cm and area 10 cm^2 . Find its length in the form $a + b\sqrt{5}$, where a and b are rational numbers.

SOLUTION

1. Length = area $\div$ width:

$$\text{length} = \frac{10}{5 - \sqrt{5}}$$

2. Rationalise with the conjugate $5 + \sqrt{5}$:

$$\frac{10}{5 - \sqrt{5}} \times \frac{5 + \sqrt{5}}{5 + \sqrt{5}} = \frac{10(5 + \sqrt{5})}{25 - 5} = \frac{50 + 10\sqrt{5}}{20}$$

3. Simplify the fraction:

$$\text{length} = \frac{5 + \sqrt{5}}{2} = \frac{5}{2} + \frac{1}{2}\sqrt{5} \text{ cm}$$

So $a = \frac{5}{2}$ and $b = \frac{1}{2}$.