

YEAR 1 MECHANICS · CHAPTER 3

Forces and Motion

Specification: Edexcel Paper 3 §8.1–8.4 — Forces and Newton's laws · AQA section R

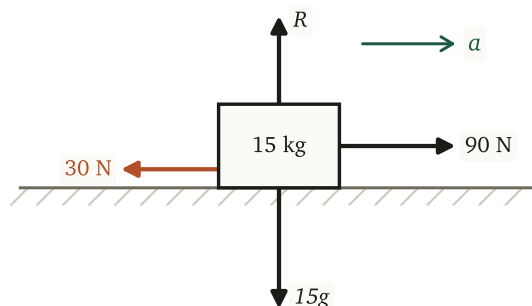
KEY FACTS — THIS CHAPTER

- **Newton's first law:** a body stays at rest or moves with constant velocity unless a resultant force acts on it. Forces in **equilibrium** have resultant zero.
- **Newton's second law:** resultant force = mass $\times$ acceleration, $F = ma$, with F in N, m in kg and a in m s^{-2} . The acceleration is in the direction of the resultant force.
- **Newton's third law:** if A pushes or pulls B , then B pushes or pulls A with a force equal in size and opposite in direction.
- Forces given as vectors add component by component, and $\mathbf{F} = m\mathbf{a}$ works with vectors too.
- **Connected particles:** use the whole system to find a , then one body on its own to find the tension (or thrust).
- A **light, inextensible** string over a **smooth** pulley has the same tension on both sides, and the particles it joins move with the same speed and the same size of acceleration.

Chapter 2 described motion without asking what causes it. This chapter adds the cause: forces. Newton's three laws connect the forces on a body to the way it accelerates. With them, and the *suvat* equations, you can predict the motion of anything from a sledge to a lift to two masses over a pulley.

1. Force diagrams and Newton's first law

Every problem starts with a **force diagram**: one body, every force acting *on* it, each drawn as an arrow in the right direction and labelled. Chapter 1 met the types: weight, normal reaction, tension, thrust and resistances. Draw the acceleration separately, with a different kind of arrow, so it is never mistaken for a force.



A 15 kg box pulled by a 90 N rope against 30 N of resistance. The acceleration arrow is drawn apart from the forces.

NEWTON'S FIRST LAW

A body remains **at rest**, or moves with **constant velocity**, unless a **resultant** force acts on it. So if a body is at rest or moving at constant velocity, the forces on it are **in equilibrium**: they balance in every direction.

The box in the diagram does not move up or down, so the vertical forces balance: $R = 15g = 147 \text{ N}$. Horizontally they don't balance, and that is what the second law is for.

2. Newton's second law

NEWTON'S SECOND LAW

$$F = ma$$

where F is the **resultant** force (in N) in the direction of the acceleration, m is the mass (in kg) and a is the acceleration (in m s^{-2}). This is where the unit comes from: 1 N is the force that gives 1 kg an acceleration of 1 m s^{-2} .

The routine is always the same. Draw the force diagram. Choose the direction of the acceleration as positive. Then write

$$(\text{forces in that direction}) - (\text{forces against it}) = ma.$$

WORKED EXAMPLE 3.1 • A BOX PULLED ALONG A FLOOR

The 15 kg box in the diagram is pulled by a horizontal rope with tension 90 N against a constant resistance of 30 N. It starts from rest. Find its acceleration, and its speed after 3 s.

Resolving horizontally, with the direction of motion positive:

$$90 - 30 = 15a \quad \Rightarrow \quad a = 4 \text{ m s}^{-2}$$

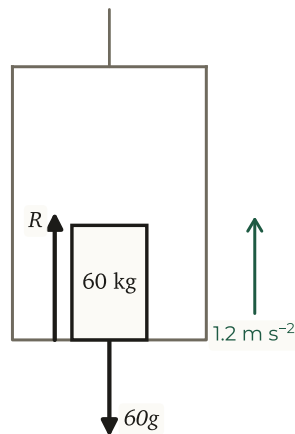
The acceleration is constant, so *suvat* applies: $v = u + at = 0 + 4(3) = 12$.

IN CONTEXT

The box accelerates at 4 m s^{-2} and is moving at 12 m s^{-1} after 3 s.

Lifts

A person standing in a lift feels the floor push up on them with the normal reaction R . When the lift accelerates, R is no longer equal to their weight, which is why your stomach lurches.



The floor pushes up with R , gravity pulls down with $60g$, and the lift accelerates upwards.

WORKED EXAMPLE 3.2 · STANDING IN A LIFT

A woman of mass 60 kg stands in a lift. Find the normal reaction from the floor when the lift (a) accelerates upwards at 1.2 m s^{-2} , (b) accelerates downwards at 1.2 m s^{-2} , and (c) moves at constant speed.

(a) Up is positive: $R - 60g = 60(1.2)$, so $R = 588 + 72 = 660 \text{ N}$.

(b) Down is positive: $60g - R = 60(1.2)$, so $R = 588 - 72 = 516 \text{ N}$.

(c) No acceleration, so the forces balance: $R = 60g = 588 \text{ N}$.

IN CONTEXT

She presses harder on the floor (660 N) when the lift speeds up going up, and less (516 N) when it speeds up going down. That's why you feel heavier and lighter.

COMMON MISTAKE

The “ ma ” side is not a force, so never draw ma on a force diagram. And F in $F = ma$ is the **resultant** force, not any single force: subtract the ones acting against the acceleration.

3. Forces as vectors

When forces are given in $\mathbf{i}$, $\mathbf{j}$ form, add them component by component to get the resultant, exactly as in Chapter 15 of Pure. Newton's second law then holds as a vector equation:

KEY RESULT

$$\mathbf{F} = m\mathbf{a}$$

The acceleration points the same way as the resultant force, and its size is $|\mathbf{a}| = \frac{|\mathbf{F}|}{m}$. Forces are in **equilibrium** when their vector sum is $\mathbf{0}$.

WORKED EXAMPLE 3.3 · A RESULTANT AS A VECTOR

A particle of mass 2 kg is acted on by the forces $(4\mathbf{i} + 2\mathbf{j}) \text{ N}$ and $(2\mathbf{i} - 10\mathbf{j}) \text{ N}$. Find its acceleration as a vector, the size of the acceleration, and the angle it makes with $\mathbf{i}$.

Resultant: $\mathbf{F} = (4 + 2)\mathbf{i} + (2 - 10)\mathbf{j} = (6\mathbf{i} - 8\mathbf{j}) \text{ N}$.

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{6\mathbf{i} - 8\mathbf{j}}{2} = (3\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-2}, \quad |\mathbf{a}| = \sqrt{3^2 + 4^2} = 5 \text{ m s}^{-2}$$

It points 3 right and 4 down, at $\tan^{-1} \frac{4}{3} = 53.1^\circ$ below $\mathbf{i}$.

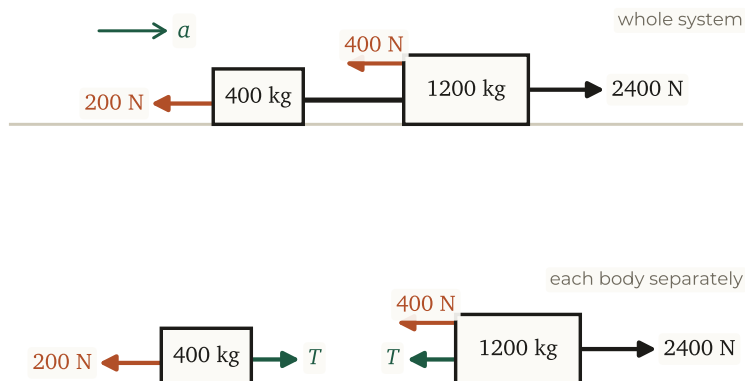
A particle that starts from rest under a constant force moves in a straight line in the direction of that force. Its velocity after time t is $\mathbf{v} = \mathbf{at}$: *suvat*, in vector form.

4. Newton's third law and connected particles

NEWTON'S THIRD LAW

When body A exerts a force on body B , body B exerts a force on A that is **equal in size** and **opposite in direction**.

A car towing a trailer is the standard example. The tow bar pulls the trailer forwards with a tension T , and by the third law it pulls the car backwards with the same T . Treat the car and trailer as **one system** and those two tensions cancel: they are *internal* forces. Pull the bodies apart to see them.



As one system the tow-bar forces cancel. Drawn separately, each body feels T .

THE CONNECTED-PARTICLES ROUTINE

1. **Whole system:** use the external forces and the total mass to find a .
2. **One body:** write $F = ma$ for the simpler body. The internal force (tension or thrust) appears, and you can solve for it.
3. **Check** with the other body.

WORKED EXAMPLE 3.4 · CAR AND TRAILER

A car of mass 1200 kg tows a trailer of mass 400 kg along a straight horizontal road. The engine produces a driving force of 2400 N. The resistances to motion are 400 N on the car and 200 N on the trailer. The tow bar is light and horizontal. Find the acceleration and the tension in the tow bar.

Whole system (1600 kg):

$$2400 - 400 - 200 = 1600a \quad \Rightarrow \quad a = \frac{1800}{1600} = 1.125 \text{ m s}^{-2}$$

Trailer only: the tension pulls it forward, the resistance holds it back:

$$T - 200 = 400(1.125) \quad \Rightarrow \quad T = 650 \text{ N}$$

Check with the car: $2400 - 400 - 650 = 1350 = 1200(1.125) \checkmark$

IN CONTEXT

The car and trailer accelerate at 1.125 m s^{-2} , and the tension in the tow bar is 650 N.

TENSION OR THRUST?

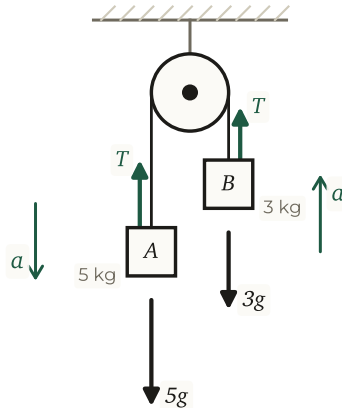
When the car **brakes**, the trailer tends to run into it, so the tow bar is squashed and *pushes*: it is in **thrust** (compression). The equations don't change. Solve for T as usual, and a negative answer means the force acts the other way: a thrust of that size. A rope can only pull, so it can never be in thrust. That is one reason tow *bars* exist.

5. Pulleys

Particles joined by a string over a pulley are connected particles too. The string changes direction at the pulley, so write a separate equation for **each particle**, each in its own direction of motion.

MODELLING ASSUMPTION

The **string is light and inextensible**: the tension is the same all along it, and both particles have the same speed and the same size of acceleration. The **pulley is smooth**, so the tension is the same on both sides of it.



Two particles over a smooth pulley. The heavier one, A, accelerates down; B accelerates up.

WORKED EXAMPLE 3.5 · TWO PARTICLES OVER A PULLEY

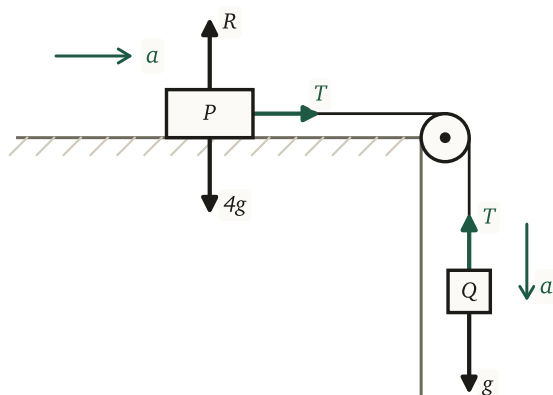
Particles A (5 kg) and B (3 kg) are joined by a light inextensible string passing over a smooth fixed pulley and released from rest with the string taut. Find the acceleration, the tension in the string and the force the string exerts on the pulley.

A is heavier, so it moves down and B moves up, each with acceleration a .

$$A \text{ (down)} : 5g - T = 5a \qquad B \text{ (up)} : T - 3g = 3a$$

Adding eliminates T : $2g = 8a$, so $a = \frac{g}{4} = 2.45 \text{ m s}^{-2}$. Then $T = 3g + 3a = 29.4 + 7.35 = 36.75$, so $T = 36.8 \text{ N}$ (3 s.f.).

The string pulls down on the pulley on both sides, with a total force of $2T = 73.5 \text{ N}$.



P (4 kg) on a smooth table, joined over the edge to Q (1 kg), which hangs freely.

WORKED EXAMPLE 3.6 · ONE ON THE TABLE, ONE HANGING

In the diagram, P has mass 4 kg and rests on a smooth horizontal table. Q has mass 1 kg. The system is released from rest. Find the acceleration and the tension.

$$P \text{ (horizontally)} : T = 4a \qquad Q \text{ (down)} : g - T = a$$

Adding: $g = 5a$, so $a = 1.96 \text{ m s}^{-2}$, and $T = 4(1.96) = 7.84 \text{ N}$.

The hanging weight g has to accelerate *both* masses, 5 kg in all, which is why a is only a fifth of g .

COMMON MISTAKE

Don't use "whole system" reasoning on a pulley by adding weights that act in different directions. Here the whole-system version only works if you think of the string as straightened out: the driving force is g and the total mass is 5 kg. The safe method is always one equation per particle, each in its own direction of motion.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution. The solutions show the full method, not just answers. Take $g = 9.8 \text{ m s}^{-2}$ throughout. Strings are light and inextensible and pulleys are smooth unless a question says otherwise.

Warm-up

Question 1

[3 marks]

- (a) A particle of mass 4 kg is acted on by a force of 20 N due east and a force of 8 N due west. Find its acceleration.
- (b) A resultant force of 15 N gives a body an acceleration of 2.5 m s^{-2} . Find its mass.

SOLUTION

(a) Resultant = $20 - 8 = 12 \text{ N}$ east, so $a = \frac{12}{4} = 3 \text{ m s}^{-2}$ due east.

(b) $m = \frac{F}{a} = \frac{15}{2.5} = 6 \text{ kg}$.

Question 2

[3 marks]

A crate of mass 12 kg rests on a horizontal floor. Draw a force diagram, find the normal reaction, and explain which of Newton's laws you used.

SOLUTION

Two forces act: the weight $12g$ vertically down and the normal reaction R vertically up.

The crate is at rest, so by **Newton's first law** the forces are in equilibrium: $R = 12g = 117.6 \text{ N}$, or 118 N (3 s.f.).

Question 3

[4 marks]

Forces $(3\mathbf{i} + 4\mathbf{j}) \text{ N}$ and $(5\mathbf{i} - 10\mathbf{j}) \text{ N}$ act on a particle of mass 2 kg. Find the resultant force, its magnitude, and the acceleration of the particle as a vector.

SOLUTION

$\mathbf{F} = (8\mathbf{i} - 6\mathbf{j}) \text{ N}$, with $|\mathbf{F}| = \sqrt{64 + 36} = 10 \text{ N}$.

$\mathbf{a} = \frac{\mathbf{F}}{2} = (4\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-2}$, which has magnitude 5 m s^{-2} .

Question 4

[3 marks]

A man of mass 80 kg stands in a lift that is accelerating upwards at 0.5 m s^{-2} . Find the normal reaction between the man and the floor.

SOLUTION

Up is positive: $R - 80g = 80(0.5)$, so $R = 784 + 40 = 824$ N.

Standard**Question 5**

[5 marks]

A box of mass 25 kg is pulled from rest across a horizontal floor by a horizontal rope. The tension in the rope is 120 N and there is a constant resistance of 45 N.

- (a) Find the acceleration of the box.
- (b) Find its speed after 4 s, and the distance it has moved by then.

SOLUTION

(a) $120 - 45 = 25a$, so $a = 3 \text{ m s}^{-2}$.

(b) $v = 0 + 3(4) = 12 \text{ m s}^{-1}$ and $s = \frac{1}{2}(3)(4^2) = 24 \text{ m}$.

Question 6

[5 marks]

A particle is in equilibrium under three forces: $(2\mathbf{i} + 5\mathbf{j})$ N, $(-7\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}$. Find $\mathbf{F}$, its magnitude, and the angle it makes with $\mathbf{i}$.

SOLUTION

In equilibrium the forces sum to $\mathbf{0}$:

$$\mathbf{F} = -[(2 - 7)\mathbf{i} + (5 + 1)\mathbf{j}] = (5\mathbf{i} - 6\mathbf{j}) \text{ N}$$

$|\mathbf{F}| = \sqrt{25 + 36} = \sqrt{61} = 7.81 \text{ N (3 s.f.)}$. It points 5 right and 6 down, at $\tan^{-1} \frac{6}{5} = 50.2^\circ$ below $\mathbf{i}$.

Question 7

[6 marks]

A car of mass 1000 kg tows a trailer of mass 500 kg along a straight horizontal road using a light horizontal tow bar. The driving force is 2250 N, and the resistances are 300 N on the car and 150 N on the trailer.

- (a) Find the acceleration.
- (b) Find the tension in the tow bar.

SOLUTION

(a) Whole system: $2250 - 300 - 150 = 1500a$, so $a = 1.2 \text{ m s}^{-2}$.

(b) Trailer: $T - 150 = 500(1.2)$, so $T = 750 \text{ N}$.

Check with the car: $2250 - 300 - 750 = 1200 = 1000(1.2) \checkmark$

Question 8

[6 marks]

A particle of mass 3 kg is at rest at the origin when a constant force $(6\mathbf{i} - 9\mathbf{j}) \text{ N}$ starts to act on it.

- (a) Find its acceleration.
- (b) Find its velocity and speed after 4 s .
- (c) Find its position vector after 4 s .

SOLUTION

(a) $\mathbf{a} = \frac{1}{3}(6\mathbf{i} - 9\mathbf{j}) = (2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-2}$.

(b) $\mathbf{v} = \mathbf{u} + \mathbf{at} = 4(2\mathbf{i} - 3\mathbf{j}) = (8\mathbf{i} - 12\mathbf{j}) \text{ m s}^{-1}$. Its speed is $\sqrt{64 + 144} = \sqrt{208} = 14.4 \text{ m s}^{-1}$ (3 s.f.).

(c) $\mathbf{r} = \mathbf{ut} + \frac{1}{2}\mathbf{at}^2 = \frac{1}{2}(16)(2\mathbf{i} - 3\mathbf{j}) = (16\mathbf{i} - 24\mathbf{j}) \text{ m}$.

Exam style**Question 9**

[9 marks]

Particles A (6 kg) and B (4 kg) are attached to the ends of a string that passes over a fixed pulley. They hang with the string taut, A 2 m above the ground and B on the ground, and are released from rest.

- (a) Find the acceleration of the particles and the tension in the string.
- (b) Find the force exerted on the pulley by the string.
- (c) Find the speed of A just before it hits the ground.
- (d) When A hits the ground it does not rebound, and B does not reach the pulley. Find the greatest height of B above the ground.

SOLUTION

(a) A (down): $6g - T = 6a$. B (up): $T - 4g = 4a$. Adding: $2g = 10a$, so $a = 1.96 \text{ m s}^{-2}$.

Then $T = 4g + 4a = 39.2 + 7.84 = 47.04$, so $T = 47.0 \text{ N}$ (3 s.f.).

(b) $2T = 94.1 \text{ N}$ (3 s.f.), vertically downwards.

(c) A falls 2 m from rest: $v^2 = 0 + 2(1.96)(2) = 7.84$, so $v = 2.8 \text{ m s}^{-1}$.

(d) At that moment B is 2 m up and rising at 2.8 m s^{-1} . The string goes slack, so B now moves freely under gravity. It rises a further s where $0 = 2.8^2 - 2(9.8)s$, so $s = \frac{7.84}{19.6} = 0.4 \text{ m}$.

IN CONTEXT

B reaches a greatest height of $2 + 0.4 = 2.4 \text{ m}$ above the ground.

Question 10

[7 marks]

A block P of mass 4 kg lies on a rough horizontal table. It is attached by a string, passing over a pulley at the edge of the table, to a particle Q of mass 3 kg, which hangs freely. The resistance to the motion of P is a constant 9.8 N. The system is released from rest.

- Write down an equation of motion for each particle.
- Find the acceleration and the tension.
- State where in your working you used the fact that the string is inextensible.

SOLUTION

(a) P (towards the pulley): $T - 9.8 = 4a$. Q (down): $3g - T = 3a$.

(b) Adding: $3g - 9.8 = 7a$, so $a = \frac{29.4 - 9.8}{7} = 2.8 \text{ m s}^{-2}$. Then $T = 9.8 + 4(2.8) = 21 \text{ N}$.

(c) In using the **same** acceleration a for both particles.

Question 11

[7 marks]

A lift of mass 500 kg carries a woman of mass 60 kg. The lift is raised by a vertical cable.

- The tension in the cable is 6000 N. Find the acceleration of the lift.
- Find the normal reaction between the woman and the floor during this motion.
- Later, the lift is moving upwards and slowing down at 1.5 m s^{-2} . Find the tension in the cable and the normal reaction on the woman.

SOLUTION

(a) Lift and woman together (560 kg), up positive: $6000 - 560g = 560a$, so $a = \frac{6000 - 5488}{560} = 0.914 \text{ m s}^{-2}$ (3 s.f.), upwards.

(b) Woman: $R - 60g = 60a$, so $R = 60(9.8 + 0.9143 \dots) = 643 \text{ N}$ (3 s.f.).

(c) Now $a = -1.5$ (up positive). Whole: $T - 560g = 560(-1.5)$, so $T = 560(8.3) = 4648$ N. Woman: $R = 60(9.8 - 1.5) = 498$ N.

IN CONTEXT

While the lift slows on the way up, the cable tension drops to 4650 N (3 s.f.) and the woman feels lighter: the floor pushes up with only 498 N instead of her weight of 588 N.

Question 12

[8 marks]

A car of mass 1200 kg tows a trailer of mass 600 kg using a light rigid tow bar. They are travelling at 20 m s^{-1} along a straight horizontal road when the car's brakes are applied. The braking force on the car is 3000 N, and there are also constant resistances of 400 N on the car and 200 N on the trailer.

- Find the deceleration.
- Find the force in the tow bar, and state whether it is a tension or a thrust.
- Find the distance travelled before the car and trailer stop.

SOLUTION

(a) Whole system, forwards positive: $-3000 - 400 - 200 = 1800a$, so $a = -2$: a deceleration of 2 m s^{-2} .

(b) Suppose the tow bar pulls the trailer forwards with tension T . Trailer: $T - 200 = 600(-2)$, so $T = -1000$.

The negative sign means the force acts the other way: the bar pushes the trailer **backwards** with 1000 N. It is a **thrust** of 1000 N.

(Check with the car: $-3000 - 400 + 1000 = -2400 = 1200(-2)$ ✓)

(c) $0 = 20^2 + 2(-2)s$, so $s = 100$ m.