

YEAR 1 MECHANICS • CHAPTER 2

Constant Acceleration

Specification: Edexcel Paper 3 §7.1–7.3, 8.3 — Kinematics • AQA section Q, R3

KEY FACTS — THIS CHAPTER

- **Displacement** s , **velocity** v and **acceleration** a are vectors: along a line, their sign gives the direction. Distance and speed are their sizes.
- On a **displacement–time** graph, the gradient is the velocity.
- On a **velocity–time** graph, the gradient is the acceleration and the **area** under it is the displacement.
- The **suvat equations**, for constant acceleration only: $v = u + at$, $s = \frac{1}{2}(u + v)t$, $s = ut + \frac{1}{2}at^2$, $s = vt - \frac{1}{2}at^2$, $v^2 = u^2 + 2as$.
- Each *suvat* equation leaves out one of the five letters. Choose the one that leaves out the letter you neither know nor want.
- Free fall: $a = g = 9.8 \text{ m s}^{-2}$ downwards. Choose a positive direction and keep every sign consistent with it.

This chapter is about motion in a straight line: a car pulling away from traffic lights, a train between stations, a ball thrown into the air. When the acceleration is constant, five short equations describe the whole motion. Every one of them comes from a single picture, the velocity–time graph.

1. The language of motion

Along a straight line, choose one direction to be **positive**. Then every vector quantity is just a number with a sign.

KEY RESULT

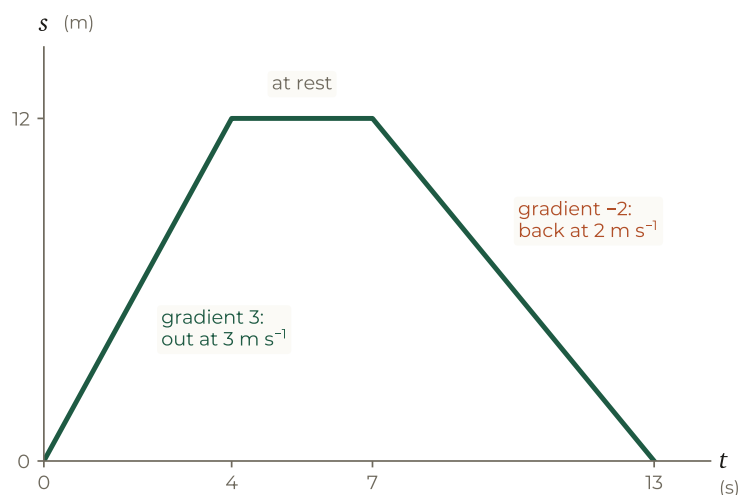
Quantity	Symbol	Unit	Its size is...
displacement from a fixed point	s	m	distance
velocity: rate of change of displacement	v	m s^{-1}	speed
acceleration: rate of change of velocity	a	m s^{-2}	

A velocity of -5 m s^{-1} means a speed of 5 m s^{-1} in the negative direction. A **deceleration** of 3 m s^{-2} is an acceleration of -3 m s^{-2} : the velocity is falling. Be careful with “slowing down”, though. Slowing down happens when v and a have *opposite* signs.

2. Displacement–time graphs

KEY RESULT

On a **displacement–time** (s – t) graph, the **gradient** is the velocity. A horizontal line means the body is at rest. A negative gradient means it is moving back towards the start.



Out at 3 m s^{-1} , a rest, then back at 2 m s^{-1} . Each gradient is a velocity.

The walker in the graph covers 24 m in total, but ends with displacement 0. The average *speed* is $\frac{24}{13} = 1.85 \text{ m s}^{-1}$. The average *velocity* is 0.

3. Velocity–time graphs

The velocity–time graph carries more information, and it is the one you’ll use most.

KEY RESULT

On a **velocity–time** (v – t) graph:

- the **gradient** is the **acceleration**;
- the **area** between the graph and the t -axis is the **displacement**. Area below the axis counts as negative displacement.

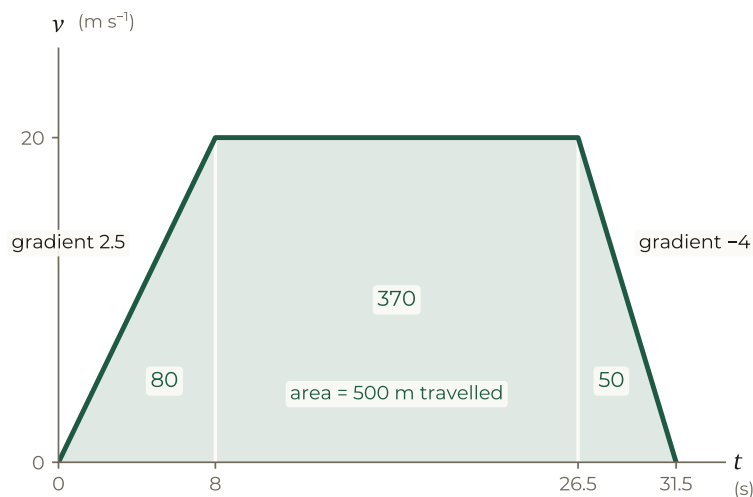
WHY THE AREA IS THE DISPLACEMENT

Over a short time δt , a body moving at velocity v travels about $v \delta t$. That is the area of a thin strip under the graph. Add up all the strips and you have the total displacement, which is exactly the area under the curve. This is the idea behind integration from Chapter 14 of Pure, and in Chapter 4 you’ll write it as $s = \int v dt$.

WORKED EXAMPLE 2.1 • A JOURNEY IN THREE STAGES

A car starts from rest and accelerates uniformly to 20 m s^{-1} in 8 s. It travels at 20 m s^{-1} for T seconds, then decelerates uniformly to rest in 5 s. The whole journey covers 500 m. Find T , the acceleration and the deceleration.

Sketch the v – t graph: a trapezium. Its area is the distance.



Three pieces of area: two triangles and a rectangle.

$$\frac{1}{2}(8)(20) + 20T + \frac{1}{2}(5)(20) = 500$$

$$130 + 20T = 500$$

$$T = 18.5$$

The gradients give the acceleration, $\frac{20}{8} = 2.5 \text{ m s}^{-2}$, and the deceleration, $\frac{20}{5} = 4 \text{ m s}^{-2}$.

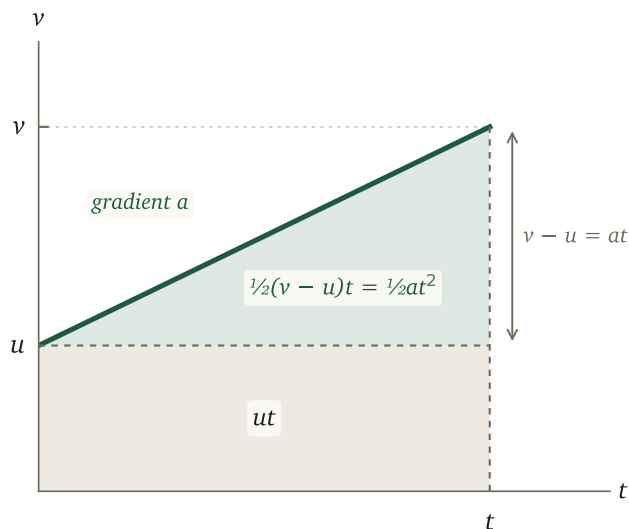
IN CONTEXT

The car travels at constant speed for 18.5 s. It accelerates at 2.5 m s^{-2} and decelerates at 4 m s^{-2} .

4. The suvat equations

When the acceleration is constant, the v - t graph is a straight line. Five quantities describe the motion over a time interval:

- s , the displacement;
- u , the initial velocity;
- v , the final velocity;
- a , the acceleration;
- t , the time.



The area under the line is a rectangle ut plus a triangle $\frac{1}{2}at^2$, or a single trapezium $\frac{1}{2}(u + v)t$.

DERIVING SUVAT FROM THE GRAPH

Gradient. The line rises by $v - u$ over time t , so $a = \frac{v - u}{t}$:

$$v = u + at$$

Area. The region under the line is a trapezium with parallel sides u and v :

$$s = \frac{1}{2}(u + v)t$$

Eliminate.

- Substitute $v = u + at$ into the area formula: $s = ut + \frac{1}{2}at^2$. This is also the rectangle ut plus the triangle $\frac{1}{2} \cdot t \cdot at$.
- Substitute $u = v - at$ instead: $s = vt - \frac{1}{2}at^2$.
- Substitute $t = \frac{v - u}{a}$: $s = \frac{(u + v)(v - u)}{2a}$, which rearranges to $v^2 = u^2 + 2as$.

Each equation leaves out exactly one of the five letters:

Equation	Leaves out
$v = u + at$	s
$s = \frac{1}{2}(u + v)t$	a
$s = ut + \frac{1}{2}at^2$	v
$s = vt - \frac{1}{2}at^2$	u
$v^2 = u^2 + 2as$	t

The routine: write down all five letters, fill in the three you know, mark the one you want, and pick the equation that doesn't need the fifth.

WORKED EXAMPLE 2.2 · CHOOSING THE RIGHT EQUATION

A car decelerates uniformly from 30 m s^{-1} to 12 m s^{-1} over a distance of 84 m . Find the deceleration and the time taken.

$s = 84$, $u = 30$, $v = 12$, $a = ?$, t not needed yet. The equation without t is $v^2 = u^2 + 2as$:

$$144 = 900 + 2a(84) \Rightarrow a = \frac{144 - 900}{168} = -4.5$$

Now t : $v = u + at$ gives $12 = 30 - 4.5t$, so $t = 4$.

IN CONTEXT

The car decelerates at 4.5 m s^{-2} and takes 4 s. The negative sign in $a = -4.5$ is the deceleration, so don't state "a deceleration of -4.5 ".

WORKED EXAMPLE 2.3 · A QUADRATIC IN T

A cyclist passes a lamp post at 4 m s^{-1} and accelerates uniformly at 1.5 m s^{-2} . How long does she take to cover the next 30 m?

$s = 30$, $u = 4$, $a = 1.5$, $t = ?$, v not needed. So use $s = ut + \frac{1}{2}at^2$:

$$30 = 4t + 0.75t^2 \quad \Rightarrow \quad 3t^2 + 16t - 120 = 0$$

$$t = \frac{-16 \pm \sqrt{16^2 + 4(3)(120)}}{6} = \frac{-16 \pm \sqrt{1696}}{6}$$

The negative root means nothing here, because the cyclist only moves forward in time from the lamp post. So $t = 4.20 \text{ s}$ (3 s.f.).

ON YOUR CALCULATOR

A polynomial or equation solver will give both roots. In an exam, still show the quadratic and the formula: the method marks are for the working.

COMMON MISTAKE

suvat only works when the acceleration is **constant**. For a journey in stages, like the car in Example 2.1, apply the equations to one stage at a time. Remember that the final velocity of one stage is the initial velocity of the next.

5. Vertical motion under gravity

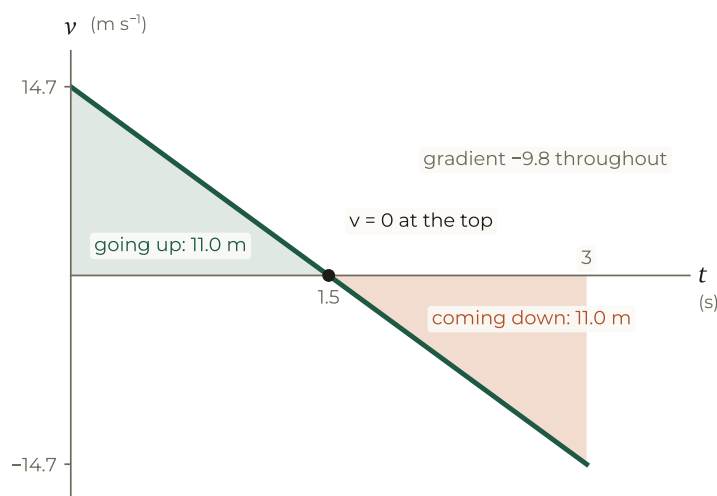
Close to the Earth's surface, and with air resistance ignored, every falling object has the same constant acceleration: $g = 9.8 \text{ m s}^{-2}$ downwards. So *suvat* applies to anything thrown straight up or dropped.

MODELLING ASSUMPTION

The object is modelled as a **particle** moving freely under gravity. Air resistance is ignored, and g is taken to be constant.

KEY RESULT

Choose **up** as positive (or down, but stick to your choice). Then $a = -9.8$, and a point *below* the starting point has negative s . At the highest point $v = 0$. A body thrown up returns to its starting level with the same speed it was thrown at.



Thrown up at 14.7 m s^{-1} : the area above the axis (going up) equals the area below it (coming down).

WORKED EXAMPLE 2.4 · THROWN STRAIGHT UP

A ball is thrown vertically upwards at 14.7 m s^{-1} from ground level. Find (a) its greatest height, (b) the time it takes to reach it, and (c) the total time it is in the air.

Take up as positive: $u = 14.7$, $a = -9.8$.

(a) At the top $v = 0$. Without t : $0 = 14.7^2 - 2(9.8)s$, so $s = \frac{216.09}{19.6} = 11.0 \text{ m}$ (3 s.f.).

(b) $v = u + at$: $0 = 14.7 - 9.8t$, so $t = 1.5 \text{ s}$.

(c) It lands when $s = 0$: $0 = 14.7t - 4.9t^2 = t(14.7 - 4.9t)$, so $t = 3 \text{ s}$. That's twice the time to the top, as the symmetry of the graph promises.

WORKED EXAMPLE 2.5 · LANDING BELOW THE START

A stone is thrown vertically upwards at 4.9 m s^{-1} from the top of a tower 29.4 m high. How long does it take to reach the ground, and how fast is it moving then?

Up is positive, so the ground is at $s = -29.4$. With $u = 4.9$, $a = -9.8$:

$$-29.4 = 4.9t - 4.9t^2 \quad \Rightarrow \quad t^2 - t - 6 = 0 \quad \Rightarrow \quad (t - 3)(t + 2) = 0$$

so $t = 3 \text{ s}$. Then $v^2 = u^2 + 2as = 4.9^2 + 2(-9.8)(-29.4) = 600.25$, so the speed is 24.5 m s^{-1} .

IN CONTEXT

The stone reaches the ground after 3 s, moving at 24.5 m s^{-1} .

COMMON MISTAKE

The most common error is a sign slip: using $s = +29.4$ for a point *below* the start, or $a = +9.8$ with up taken as positive. Write “up is positive” at the start of every vertical-motion answer and check each sign against it.

Practice

Twelve questions in three tiers. Try each one properly before opening the solution. The solutions show the full method, not just answers. Take $g = 9.8 \text{ m s}^{-2}$ throughout.

Warm-up**Question 1**

[3 marks]

A car accelerates uniformly from 5 m s^{-1} to 25 m s^{-1} in 8 s. Find its acceleration and the distance it travels in that time.

SOLUTION

$$a = \frac{v - u}{t} = \frac{25 - 5}{8} = 2.5 \text{ m s}^{-2}.$$

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}(5 + 25)(8) = 120 \text{ m}.$$

Question 2

[3 marks]

A particle moving at 18 m s^{-1} decelerates uniformly and comes to rest in 6 s. Find the deceleration and the distance travelled while stopping.

SOLUTION

$$0 = 18 + 6a, \text{ so } a = -3: \text{ a deceleration of } 3 \text{ m s}^{-2}.$$

$$s = \frac{1}{2}(18 + 0)(6) = 54 \text{ m}.$$

Question 3

[4 marks]

A cyclist starts from rest and accelerates uniformly to 8 m s^{-1} in 4 s. She rides at 8 m s^{-1} for 10 s, then decelerates uniformly to rest in 6 s.

- (a) Sketch the velocity–time graph.
- (b) Find the total distance travelled and her average speed.

SOLUTION

(a) A trapezium: a line rising from (0, 0) to (4, 8), flat to (14, 8), then falling to (20, 0).

(b) The distance is the area: $\frac{1}{2}(4)(8) + 10(8) + \frac{1}{2}(6)(8) = 16 + 80 + 24 = 120 \text{ m}$.

$$\text{Average speed} = \frac{120}{20} = 6 \text{ m s}^{-1}.$$

Question 4

[3 marks]

A stone is dropped from rest off a bridge and hits the water 2.5 s later. Find the height of the bridge above the water and the speed at which the stone hits it.

SOLUTION

Take down as positive: $u = 0$, $a = 9.8$, $t = 2.5$.

$$s = ut + \frac{1}{2}at^2 = \frac{1}{2}(9.8)(2.5)^2 = 30.625, \text{ so the height is } 30.6 \text{ m (3 s.f.)}.$$

$$v = u + at = 9.8 \times 2.5 = 24.5 \text{ m s}^{-1}.$$

Standard**Question 5**

[4 marks]

A car passes a point A at 8 m s^{-1} and accelerates uniformly at 2 m s^{-2} until it passes B , 84 m further on. Find its speed at B and the time it takes to get from A to B .

SOLUTION

$u = 8$, $a = 2$, $s = 84$. Without t : $v^2 = 8^2 + 2(2)(84) = 64 + 336 = 400$, so $v = 20 \text{ m s}^{-1}$.

Then $v = u + at$: $20 = 8 + 2t$, so $t = 6 \text{ s}$.

Question 6

[5 marks]

A ball is thrown vertically upwards at 19.6 m s^{-1} from ground level. Find:

- (a) the greatest height it reaches;
- (b) the time it takes to reach that height;
- (c) the total time it is in the air.

SOLUTION

Up is positive: $u = 19.6$, $a = -9.8$.

(a) At the top $v = 0$: $0 = 19.6^2 - 2(9.8)s$, so $s = \frac{384.16}{19.6} = 19.6 \text{ m}$.

(b) $0 = 19.6 - 9.8t$, so $t = 2 \text{ s}$.

(c) $s = 0$: $19.6t - 4.9t^2 = 0 \Rightarrow t(19.6 - 4.9t) = 0 \Rightarrow t = 4 \text{ s}$.

Question 7

[5 marks]

A particle passes a point O with velocity 10 m s^{-1} and moves with constant acceleration -2 m s^{-2} .

- (a) Find the times at which its displacement from O is 16 m , and explain why there are two.
- (b) Find its greatest displacement from O .

SOLUTION

(a) $16 = 10t - t^2 \Rightarrow t^2 - 10t + 16 = 0 \Rightarrow (t - 2)(t - 8) = 0$, so $t = 2$ or $t = 8$.

The particle passes the 16 m point on its way out ($t = 2$), carries on until it stops, then comes back through the same point ($t = 8$).

(b) It is furthest away when $v = 0$: $0 = 10 - 2t \Rightarrow t = 5$. Then $s = 10(5) - 5^2 = 25 \text{ m}$.

Question 8

[6 marks]

A train starts from rest and accelerates uniformly to a speed $V \text{ m s}^{-1}$ in 40 s. It travels at that speed for 3 minutes, then decelerates uniformly to rest in 60 s. The total distance is 5.75 km.

- (a) Sketch the velocity–time graph.
- (b) Find V .
- (c) Find the acceleration and the deceleration.

SOLUTION

(a) A trapezium: up to V over the first 40 s, level for 180 s, then down to 0 over 60 s (280 s in all).

(b) Work in metres and seconds. The area is

$$\frac{1}{2}(40)V + 180V + \frac{1}{2}(60)V = 230V = 5750 \Rightarrow V = 25$$

(c) Acceleration = $\frac{25}{40} = 0.625 \text{ m s}^{-2}$. Deceleration = $\frac{25}{60} = 0.417 \text{ m s}^{-2}$ (3 s.f.).

Exam style**Question 9**

[7 marks]

A car A is travelling along a straight road at a constant 20 m s^{-1} . As it passes a point P , a motorbike B sets off from rest at P in the same direction, accelerating uniformly at 2.5 m s^{-2} .

- (a) Find the time taken for B to catch up with A , and the distance from P at which it does.
- (b) Find the speed of B as it catches A .
- (c) Find the greatest distance between them before B catches up.

SOLUTION

(a) After t seconds, $s_A = 20t$ and $s_B = \frac{1}{2}(2.5)t^2 = 1.25t^2$.

They are level when $20t = 1.25t^2$, i.e. $t(1.25t - 20) = 0$, so $t = 16 \text{ s}$ (the root $t = 0$ is the start).
Distance = $20 \times 16 = 320 \text{ m}$.

(b) $v_B = 0 + 2.5 \times 16 = 40 \text{ m s}^{-1}$.

(c) The gap is $d = 20t - 1.25t^2$. It grows while A is faster and shrinks once B is faster, so it is greatest when their speeds are equal: $2.5t = 20 \Rightarrow t = 8$. Then $d = 160 - 80 = 80 \text{ m}$.

IN CONTEXT

B catches A after 16 s, 320 m from P , moving at 40 m s^{-1} . Before that, the greatest gap is 80 m, 8 s after the start.

Question 10

[7 marks]

A stone is thrown vertically upwards at 14 m s^{-1} from the edge of a cliff, 21 m above the sea.

- (a) Find the time it takes to reach the sea.
- (b) Find the speed with which it hits the sea.
- (c) Find the length of time for which the stone is at least 5 m above the point of projection.

SOLUTION

Up is positive, so the sea is at $s = -21$. $u = 14$, $a = -9.8$.

(a) $-21 = 14t - 4.9t^2 \Rightarrow 4.9t^2 - 14t - 21 = 0$.

$$t = \frac{14 + \sqrt{14^2 + 4(4.9)(21)}}{9.8} = \frac{14 + \sqrt{607.6}}{9.8} = 3.94 \text{ s (3 s.f.)}$$

(The other root is negative.)

(b) $v^2 = 14^2 + 2(-9.8)(-21) = 607.6$, so the speed is 24.6 m s^{-1} (3 s.f.).

(c) $5 = 14t - 4.9t^2 \Rightarrow 4.9t^2 - 14t + 5 = 0$, so

$$t = \frac{14 \pm \sqrt{196 - 98}}{9.8} = \frac{14 \pm \sqrt{98}}{9.8}$$

The stone is above 5 m between these two times, for $\frac{2\sqrt{98}}{9.8} = 2.02 \text{ s (3 s.f.)}$.

Question 11

[6 marks]

A particle P moves along a straight line. At $t = 0$ it passes O with velocity 6 m s^{-1} , and it has a constant acceleration of -2 m s^{-2} .

- (a) Find the velocity of P when $t = 5$.
- (b) Find the displacement of P from O when $t = 5$.
- (c) Find the total distance P travels in the first 5 seconds.

SOLUTION

(a) $v = 6 - 2(5) = -4 \text{ m s}^{-1}$: P is now moving back towards O .

(b) $s = 6(5) - 5^2 = 5 \text{ m}$.

(c) P turns round when $v = 0$: $6 - 2t = 0 \Rightarrow t = 3$, where $s = 18 - 9 = 9 \text{ m}$. So it goes 9 m out, then comes back from 9 m to 5 m, another 4 m. The total distance is $9 + 4 = 13 \text{ m}$.

On the v - t graph, that's a triangle of area 9 above the axis and one of area 4 below it: displacement $9 - 4 = 5$, distance $9 + 4 = 13$.

Question 12

[7 marks]

- (a) Use a velocity–time graph to show that a body with initial velocity u and constant acceleration a has displacement $s = ut + \frac{1}{2}at^2$ after time t .
- (b) Hence show that the distance travelled in the n th second of motion is $u + a\left(n - \frac{1}{2}\right)$.
- (c) A particle moving with constant acceleration travels 13 m in the 4th second and 17 m in the 6th second. Find its initial velocity and its acceleration.

SOLUTION

(a) The graph is a straight line from $(0, u)$ to $(t, u + at)$. The area under it is the rectangle $u \times t$ plus a triangle with base t and height at :

$$s = ut + \frac{1}{2} \cdot t \cdot at = ut + \frac{1}{2}at^2 \quad \blacksquare$$

(b) The n th second runs from $t = n - 1$ to $t = n$:

$$\left(un + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right) = u + \frac{1}{2}a(2n-1)$$

$$= u + a\left(n - \frac{1}{2}\right) \quad \blacksquare$$

(c) $u + 3.5a = 13$ and $u + 5.5a = 17$. Subtracting, $2a = 4$, so $a = 2$ and $u = 6$.

IN CONTEXT

The particle starts at 6 m s^{-1} and accelerates at 2 m s^{-2} .